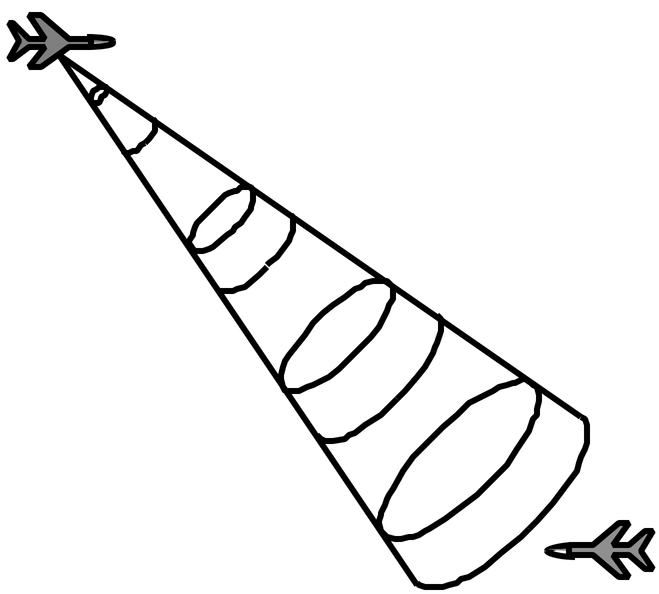
Microwave Devices & Radar

LECTURE NOTES
VOLUME II

by Professor David Jenn

Contributors: Professors F. Levien, G. Gill, J. Knorr, and J. Lebaric



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Other Sources of Loss

There are many sources of loss that are not accounted for in the basic form of the radar range equation. Similarly, there are several signal processing methods available to increase the effective signal level. Thus general loss and gain factors are added to the RRE:

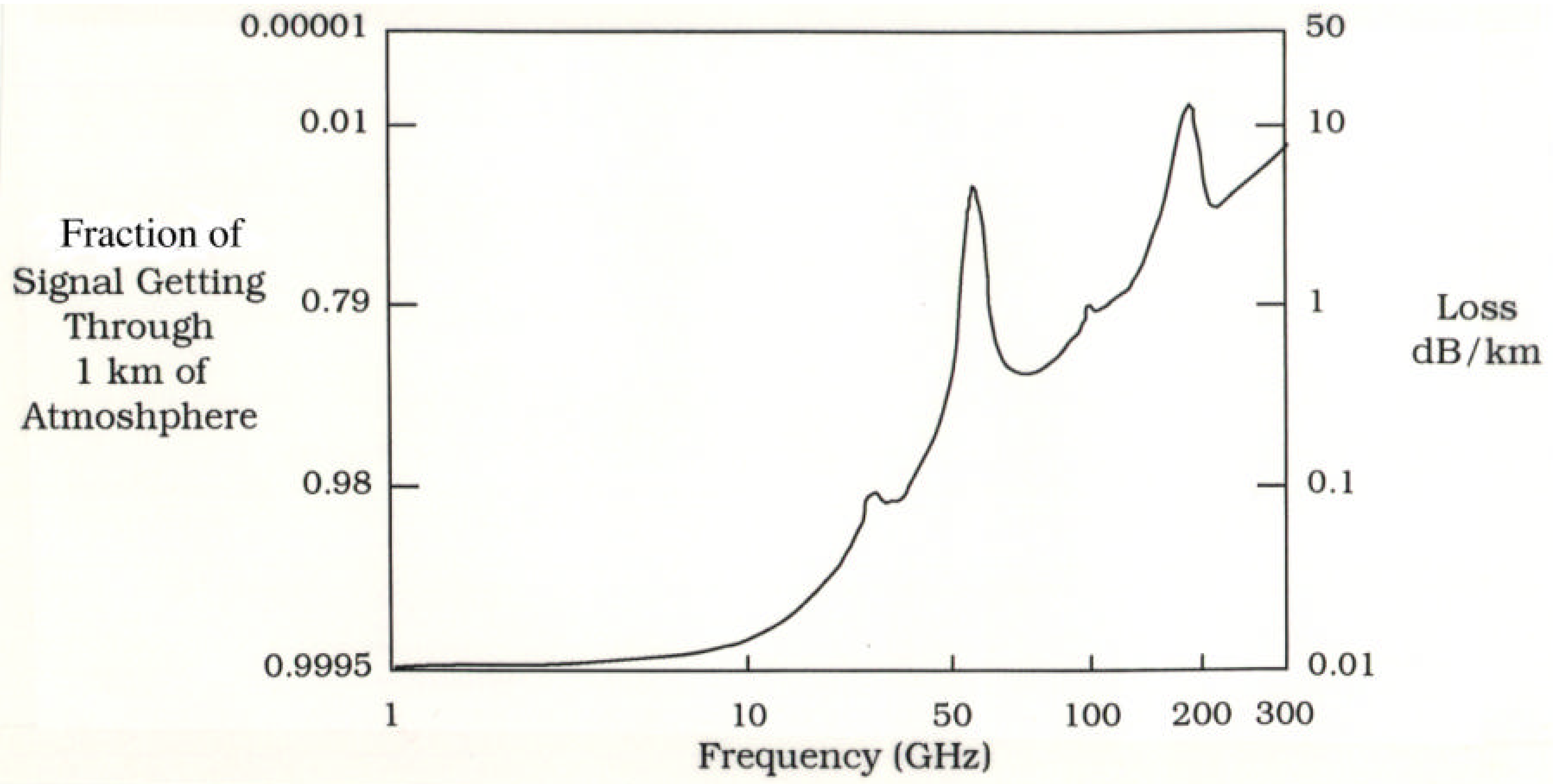
$$\mathrm{SNR} = \frac{P_r}{N_o} = \frac{P_t G_t G_r \sigma \lambda^2 G_p}{(4\pi)^3 R^4 k \mathrm{T}_s B_n L}$$

L is a "catch all" loss factor (>1). A fraction of the loss can be attributed to each source if its contribution is known. For instance, $L = L_b L_B L_x L_a L_c$ where the loss sources are:

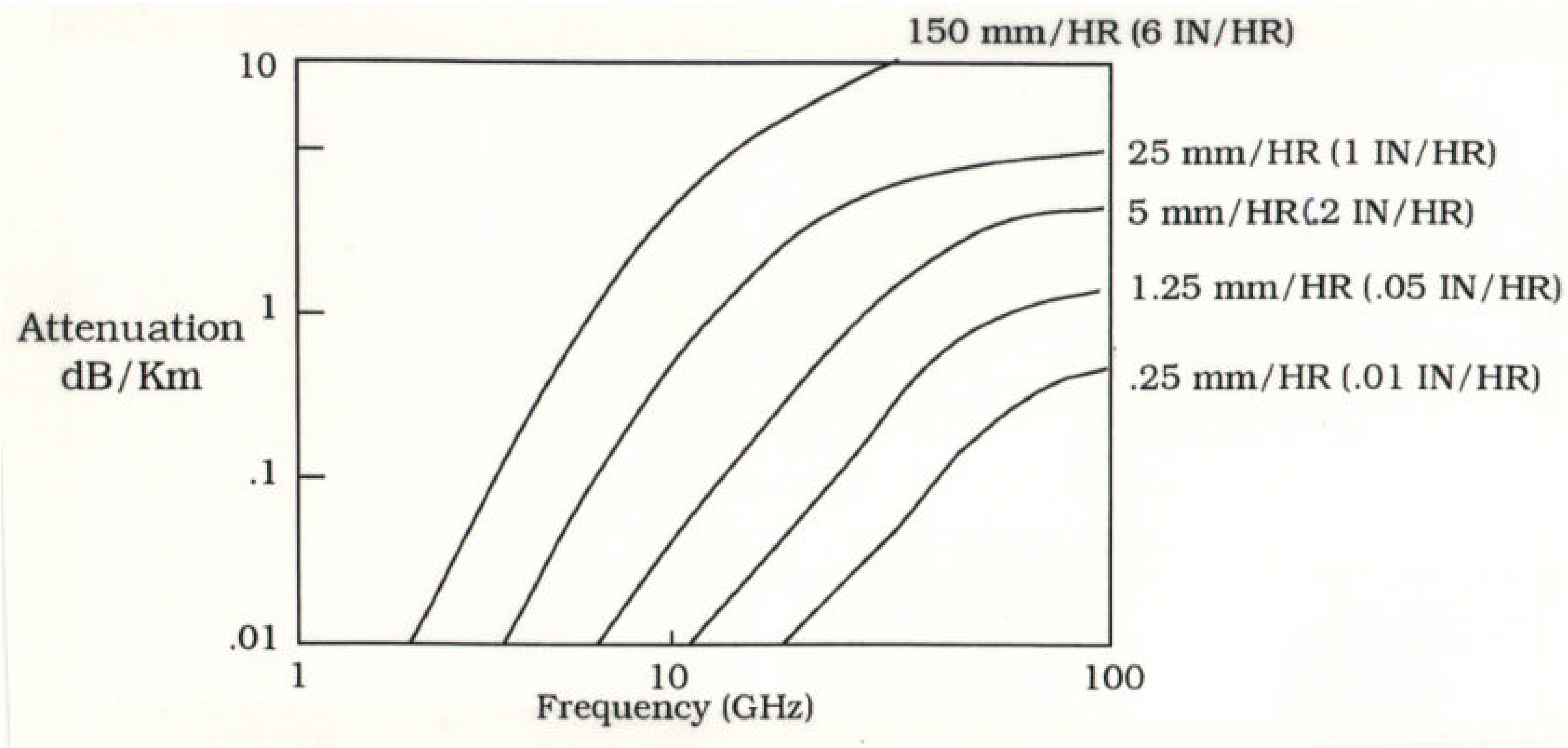
1. transmission line loss, L_x
2. atmospheric attenuation and rain loss, L_a ($1/L_a = e^{-2\alpha R}$, α = one-way power attenuation coefficient)
3. secondary background noise and interference sources, L_b
4. antenna beamshape loss, L_B
5. collapsing loss, L_c

G_p is the processing gain which can be achieved by integration and various correlation methods.

Atmospheric Attenuation



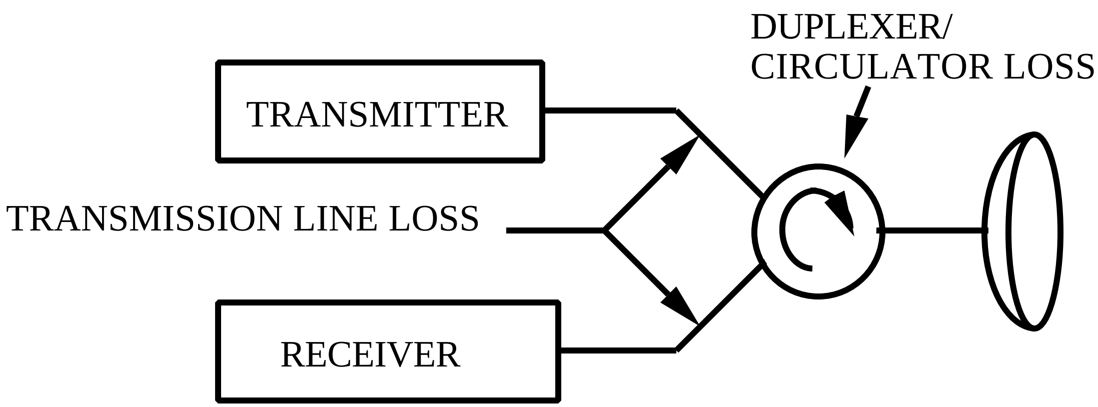
Rain Attenuation



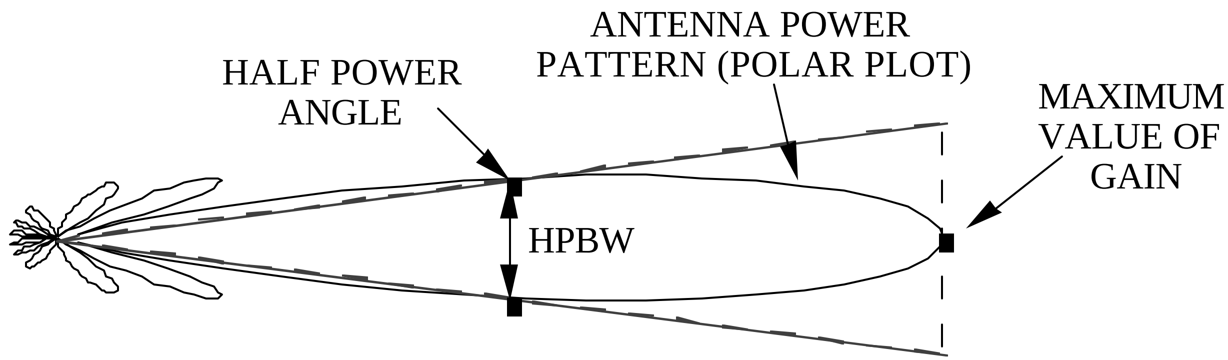
Transmission Line Loss

Transmission lines between the antenna and receiver and transmitter can have significant losses. Traditionally these have been called plumbing loss because the primary contributor was long sections of waveguide. Sources of loss include:

1. cables and waveguide runs (0.25 to 1 dB per meter)
2. devices have insertion loss
   duplexer, rotary joints, filters, switches, etc.
3. devices and connectors have mismatch loss (VSWR $\neq$ 1)



Antenna Beamshape Loss

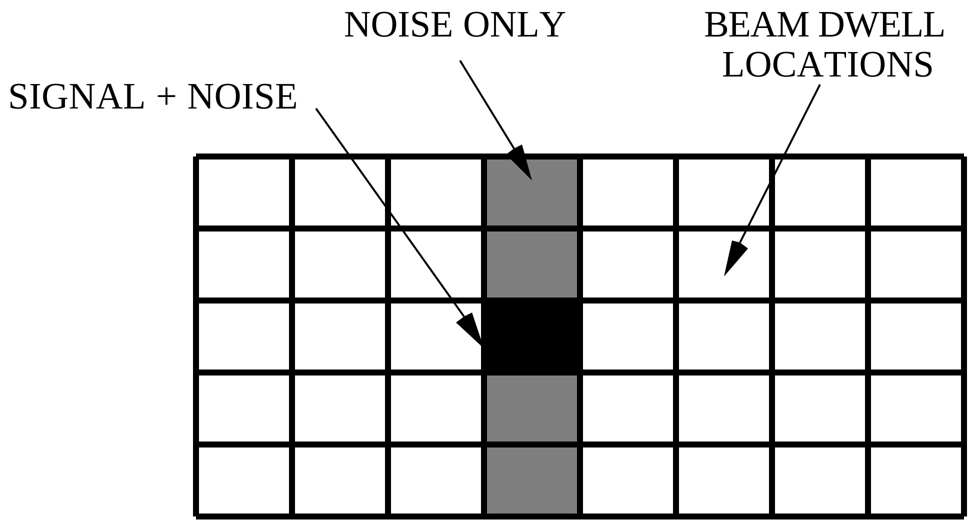


In the form of the RRE with pulse integration, constant gain has been assumed for all pulses (the duration of t_{ot}). The gain is actually changing with scan across the target. The result is lower SNR that with the approximate antenna model. The beamshape loss for a Gaussian beam, $G(q) = G_o \exp[-2.773(q/q_B)^2]$, when integrating n (odd) pulses is

$$L_B \approx \frac{n}{1 + 2\sum_{m=1}^{(n-1)/2} e^{-5.55m^2/(n_B - 1)^2}}$$

per dimension of beamshape. (For example, a "fan beam" is one dimensional). The loss tends to 1.6 dB for a large number of pulses.

Collapsing Loss



Collapsing loss can arise from several sources:

- the outputs from several receivers are added when only one contains the signal
- the outputs from several antenna beams are combined when only one contains signal

The effect is the same as adding extra noise pulses, say m, in which case the collapsing loss can be defined as

$$\frac{1}{L_c} = \frac{(n \text{ signal plus noise channels } + \ m \text{ noise only channels})}{(n \text{ signal plus noise channels})} = \frac{(\mathrm{SNR})_{m+n}}{(\mathrm{SNR})_n}$$

Noise Figure & Effective Temperature (1)

Definition of noise figure:

$$F_n = \frac{(S/N)_{\text{in}}}{(S/N)_{\text{out}}} = \frac{S_{\text{in}}/N_{\text{in}}}{S_{\text{out}}/N_{\text{out}}} = \frac{N_{\text{out}}}{k\mathrm{T}_o B_n G}$$

where $G = \dfrac{S_{\text{out}}}{S_{\text{in}}}$. By convention, noise figure is defined at the <u>standard temperature</u> of $\mathrm{T}_o = 290$ K. The noise out is the amplified noise in plus the noise added by the device

$$F_n = \frac{GN_{\text{in}} + \Delta N}{k\mathrm{T}_o B_n G} = 1 + \frac{\Delta N}{k\mathrm{T}_o B_n G}$$

ΔN can be viewed as originating from an increase in temperature. The <u>effective temperature</u> is

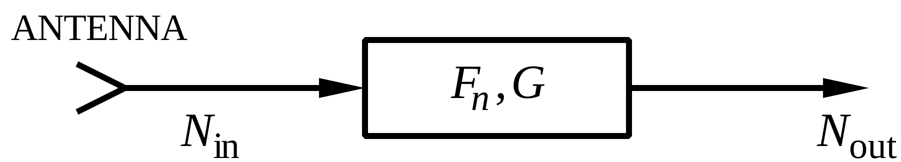
$$F_n = 1 + \frac{k\mathrm{T}_e B_n G}{k\mathrm{T}_o B_n G} = 1 + \frac{\mathrm{T}_e}{\mathrm{T}_o}$$

Solve for effective temperature in terms of noise figure

$$\mathrm{T}_e = (F_n - 1)\mathrm{T}_o$$

Comments on Noise Figure & Temperature

- Originally the term <u>noise figure</u> referred to the dB value and <u>noise factor</u> to the numeric value. Today both terms are synonymous.
- Noise figure is only unique when the input noise level is defined. It must always be reduced to a number that is proportional to noise temperature to be used in calculations.
- If the antenna temperature is unknown it is usually assumed that $\mathrm{T}_A = \mathrm{T}_o$



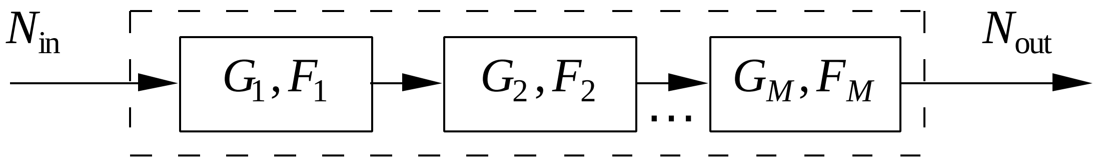
$$N_{\text{in}} = k\mathrm{T}_A B_n$$

$$N_{\text{out}} = \underbrace{k\mathrm{T}_A B_n G}_{\text{INPUT NOISE AT OUTPUT}} + \underbrace{k\mathrm{T}_e B_n G}_{\text{NOISE ADDED BY THE DEVICE}} \underset{\mathrm{T}_A = \mathrm{T}_o}{\Rightarrow} k B_n G(\mathrm{T}_o + \mathrm{T}_e) = k\mathrm{T}_o B_n G\left(1 + \frac{\mathrm{T}_e}{\mathrm{T}_o}\right) = k\mathrm{T}_o B_n G F_n$$

Thus, if $\mathrm{T}_A = \mathrm{T}_o$ then $k\mathrm{T}_o B_n F_n$ can be substituted for $k\mathrm{T}_s B_n$ in the RRE.

Noise in Cascaded Networks (1)

Examine how the noise figure is affected when M devices are cascaded



Note:
1. same B_n for each stage
2. total gain is $G = G_1 G_2 \cdots G_M$
3. denote the overall noise figure as F_o
4. devices are impedance matched

Noise from the first amplifier: $\left[(F_1 - 1)k\mathrm{T}_o B_n\right] G_1 G_2 \cdots G_M$

Noise from the second amplifier: $\left[(F_2 - 1)k\mathrm{T}_o B_n\right] G_2 G_3 \cdots G_M$

Extending to M stages:

$$N_{\text{out}} = k\mathrm{T}_o B_n \left\{ \prod_{m=1}^{M} G_m + (F_1 - 1) G_1 \prod_{m=2}^{M} G_m + \cdots + (F_M - 1) G_M \right\}$$

But N_{out} has the form $k\mathrm{T}_o B_n F_o G$. Comparing with the above,

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_M - 1}{G_1 G_2 \cdots G_{M-1}}$$

Noise Figure & Effective Temperature (2)

Summary of noise figure and effective temperature for cascaded networks:

The overall noise figure for M cascaded devices with noise figures $F_1, F_2, \ldots, F_M$ and gains $G_1, G_2, \ldots, G_M$ is

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_M - 1}{G_1 G_2 \cdots G_{M-1}}$$

The overall effective temperature for M cascaded devices with temperatures $\mathrm{T}_1, \mathrm{T}_2, \ldots, \mathrm{T}_M$ and gains $G_1, G_2, \ldots, G_M$ is

$$\mathrm{T}_e = \mathrm{T}_1 + \frac{\mathrm{T}_2}{G_1} + \frac{\mathrm{T}_3}{G_1 G_2} + \cdots + \frac{\mathrm{T}_M}{G_1 G_2 \cdots G_{M-1}}$$

Noise Figure From Loss

1. Transmission line: The fraction of electric field incident on a transmission line of length d that is transmitted is given by

$$G_x = \frac{1}{L_x} = e^{-2ad} \leq 1$$

where a is the attenuation constant. (The factor of two in the exponent is due to the fact that a is a voltage attenuation constant.) Therefore, $F_n = L_x = e^{2ad} \geq 1$.

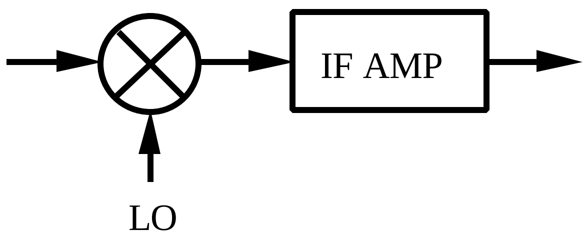
2. Mixer: Conversion loss for a mixer is

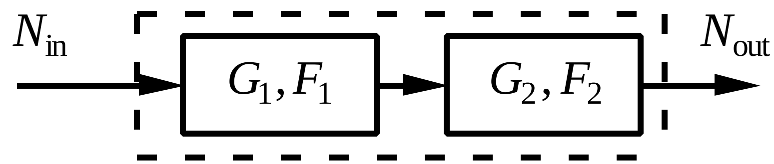
$$L_c = \frac{\text{RF power in}}{\text{IF power out}} \geq 1$$

Typical values are 4 to 6 dB. If a mixer is considered as a simple lossy two-port network (input at the carrier frequency; output at the IF frequency), then a commonly used approximation is $G_c = 1/L_c$ and $F_n = L_c$.

Examples (1)

1. A radar with the following parameters requires SNR = 10 dB for a target RCS of $5\ \mathrm{m}^2$: antenna gain = 30 dB, $P_t = 200$ kW, $f = 10$ GHz, $\tau = 1\,\mu$s, $\mathrm{T}_A = 200$ K
   receiver mixer: 10 dB conversion loss and 3 dB noise figure
   IF amplifier: 6 dB noise figure





$$F_o = F_1 + \frac{F_2 - 1}{G_1} = 2 + \frac{4-1}{0.1} = 32$$

We need the system noise temperature $\mathrm{T}_s = \mathrm{T}_e + \mathrm{T}_A = 31\mathrm{T}_o + \mathrm{T}_A = 9190$ K. Thus $k\mathrm{T}_s B_n = 1.3\times 10^{-13}$ W. It has been assumed that $B_n \approx 1/\tau$. Now $N_{\mathrm{out}} = k\mathrm{T}_s B_n G_1 G_2$ so that

$$\mathrm{SNR} = 10 = \frac{P_t G^2 \sigma \lambda^2 G_1 G_2}{(4\pi)^3 R^4 N_{\mathrm{out}} L} = \frac{P_t G^2 \sigma \lambda^2}{(4\pi)^3 R^4 k \mathrm{T}_s B_n L}$$

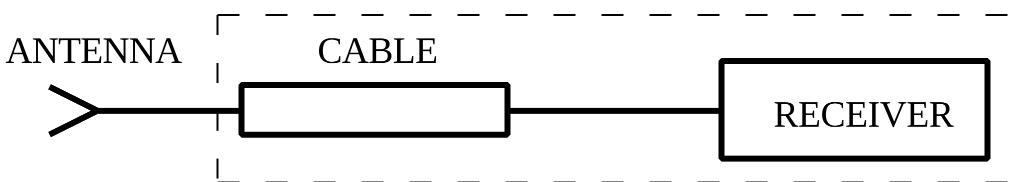
(L is for system losses such as beamshape loss, collapsing loss, etc., of which we have no information.)

Examples (2)

$$R^4 = \frac{(2\times 10^5)(10^3)^2(5)(0.03)^2}{(4\pi)^3(10)(1.3\times 10^{-13})} = 3.48\times 10^{17}$$

or $R = 24303$ m ≈ 15 miles. The range can be increased by using a low-noise amplifier (LNA) before the mixer.

2. Consider the radar receiver shown below:



antenna temperature, $T_A = 150$ K
receiver effective temperature, $T_{eR} = 400$ K
cable loss: 6 dB at 290 K

noise figure of the receiver: $F_{nR} = 1 + \frac{T_{eR}}{T_o} = 2.38$

noise figure of the cable: $F_{nL} = \frac{1}{G_L} = L = 4$

Examples (3)

noise figure of the dashed box: $F_n = F_{nL} + \dfrac{F_{nR} - 1}{G_L} = 4 + \dfrac{1.38}{0.25} = 9.52$

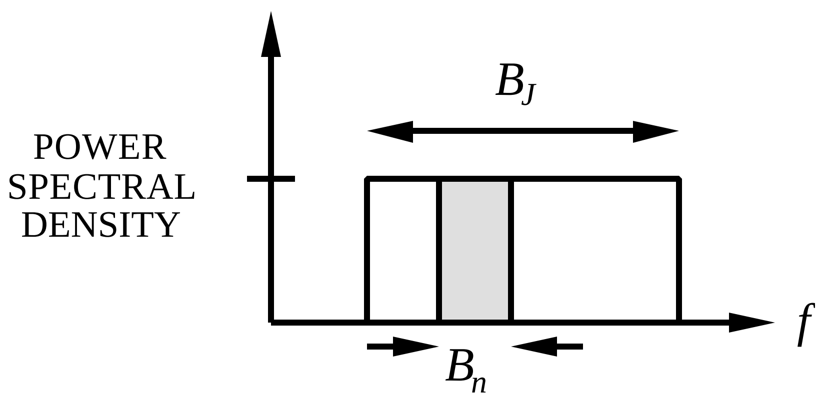
effective noise temperature of the dashed box:

$$\mathrm{T}_e = (F_n - 1)\,\mathrm{T}_o = (8.52)(290) = 2471 \text{ K}$$

Total system noise temperature:

$$\mathrm{T}_s = \mathrm{T}_A + \mathrm{T}_e = 2621 \text{ K}$$

3. We calculate the equivalent noise temperature of a wideband jammer operating against a narrowband radar



Examples (4)

The radar only sees a fraction of the jammer's radiated power

$$P_J\left(\frac{B_n}{B_J}\right)$$

The jammer can be modeled as a noise source at temperature T_J

$$N_o \equiv P_{rJ} = k\,\mathrm{T}_J B_n$$

From our earlier result the jammer power received by the radar is

$$P_{rJ} = \frac{P_J G_J G(q_J)\, l^2}{(4pR_J)^2}\left(\frac{B_n}{B_J}\right)$$

which gives an equivalent jammer temperature of

$$\mathrm{T}_J = \frac{P_{rJ}}{kB_n} = \frac{P_J G_J G(q_J)\, l^2}{(4p\ R_J)^2 k\, B_J}$$

This temperature is used in the radar equation to access the impact of jammer power on the radar’s SNR.

Examples (5)

4. Pulse radar with the following parameters:

$T_{fa} = 30$ days, $B_n = 1$ MHz, $t = 1\ m$sec, $l = 0.1$ m, $G = 30$ dB = 1000,
$q_{B_{el}} = 34^\circ$, $q_{B_{az}} = 1.2^\circ$, $P_d = 0.95$ for $s = 10$ m^2 and R=100 mi (=161 km)
$F = 5.8$ dB for the receiver, $T_A = 100$ K

(a) Peak power required to achieve $P_d = 0.95$ on a single hit basis

$$P_{fa} = \frac{1}{B_n T_{fa}} = \frac{10^{-6}}{(30)(24)(3600)} = 3.86 \times 10^{-13}$$

From Fig. 2.6, $(S/N)_1 = 16.3\text{ dB} = 42.7$

$$P_t = \frac{(4p)^2 k (T_A + T_e) B_n (S/N)_{\min} R_{\max}^4}{G A_e s}$$

$$= \frac{(4p)^2 (1.38 \times 10^{-23})(100 + 812)(10^6)(42.7)(1.61 \times 10^5)^4}{(1000)(0.8)(10)}$$

$$P_t = 0.715 \times 10^7 \text{ W}$$

Examples (6)

(b) PRF for 100 mi unambiguous range

$$f_p = \frac{c}{2R_u} = \frac{3\times10^8}{2(1.61\times10^5)} = 932\,\text{Hz}$$

(c) Number of pulses that must be integrated noncoherently for a 10 dB improvement in SNR (i.e., reduce the peak power requirement by 10 dB)

$$I_i = 10 \text{ dB} = 10$$

From Fig. 2.7(a) for, $P_d = 0.95$ and $n_f = 1/P_{fa} = 2.59\times10^{12}$ the number of pulses is

$$n_B \approx 15$$

(d) Maximum antenna scan rate if $I_i = 10$ with a PRF of 800 Hz

$$w_s = \frac{q_B f_p}{n_B} = \frac{(1.2)(800)}{15} = 64^\circ/\text{sec} = 10.7\text{rpm}$$

Examples (7)

(e) Effective temperature of the receiver

$$T_e = (F-1)T_o = (3.8-1)(290) = 814\,\mathrm{K}$$

(f) If F is reduced 3 dB the increase in maximum detection range is as follows:

$$F = 2.8\ \mathrm{dB}$$

$$T_e = (0.9)(290) = 262\ \mathrm{K}$$

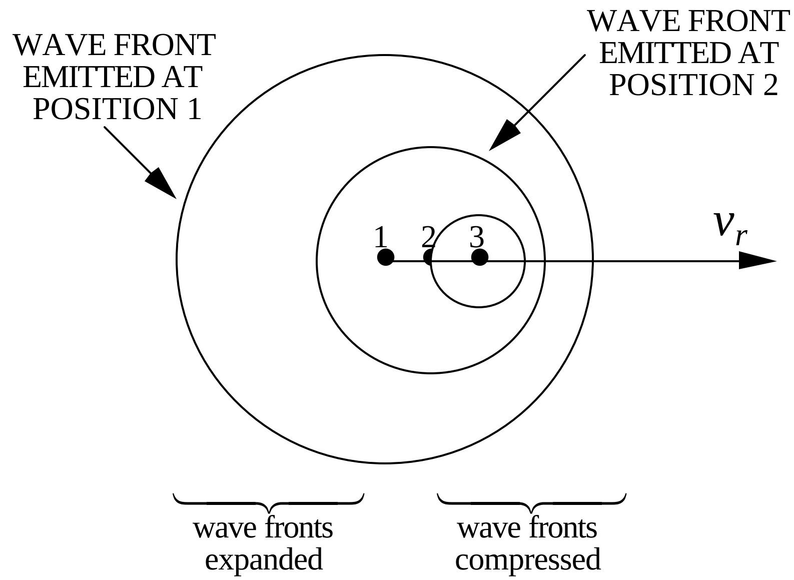
$$R_{\max}^4 \propto \frac{1}{T_A + T_e}$$

$$\frac{R_{\max_2}}{R_{\max_1}} = \left(\frac{914}{362}\right)^{1/4}$$

$$R_{\max_2} = 1.26 R_{\max_1} = 126\ \text{ miles}$$

Doppler Frequency Shift (1)

Targets in motion relative to the radar cause the return signal frequency to be shifted as shown below



The time-harmonic transmitted electric field has the form $|\vec{E}_t| \propto \cos(w_c t)$. The received signal has the form $|\vec{E}_s| \propto \cos(w_c t - 2kR)$, where the factor 2 arises from the round trip path delay. Define $\Phi(t) = -2kR$ and $R = R_o + v_r t$ (v_r is the radial component of the relative velocity vector).

Doppler Frequency Shift (2)

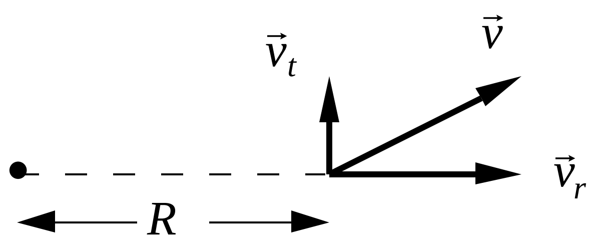
The Doppler frequency shift is given by:

$$w_d = \frac{d\Phi(t)}{dt} = \frac{d}{dt}(-2kR) = -2k\frac{dR}{dt} = -2k\,v_r$$

or in Hertz, $f_d = -\frac{2v_r}{l}$. Rewrite the signal phase as $\Phi(t) = -2k\left[R_o - \left(-\frac{w_d}{2k}\right)t\right]$ so that

$$\left|\vec{E}_s\right| \propto \cos\left[(w_c + w_d)t - 2kR_o\right]$$

A Doppler shift only occurs when the relative velocity vector has a radial component. In general there will be both radial and tangential components to the velocity:



$$R \text{ decreasing } \Rightarrow \frac{dR}{dt} < 0 \Rightarrow f_d > 0 \text{ (closing target)}$$

$$R \text{ increasing } \Rightarrow \frac{dR}{dt} > 0 \Rightarrow f_d < 0 \text{ (receeding target)}$$

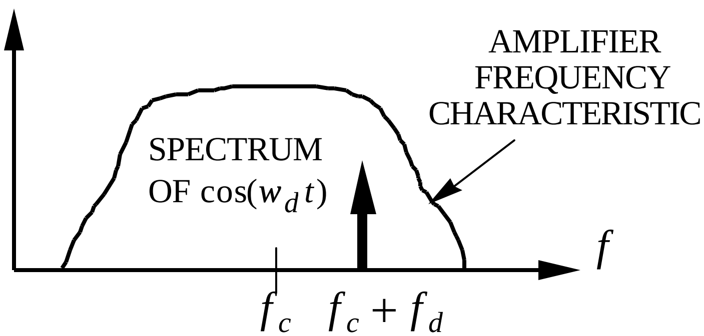
Doppler Frequency Shift (3)

The echo returned from the target varies as $|\vec{E}_s| \propto \cos[(w_c \pm w_d)t - 2kR_o]$, where the + sign is used for closing targets and - for receding targets. However, mixing causes the sign to be lost. For example, mixing the return with the carrier (homodyning) gives

$$\cos(w_c t \pm w_d t)\cos(w_c t) = \frac{1}{2}\left[\cos(\pm w_d t) + \cos(2w_c t \pm w_d t)\right]$$

Although the second term in brackets contains the sign information it is too high to be of use in a narrowband radar. The sign can be recovered using I and Q channels.

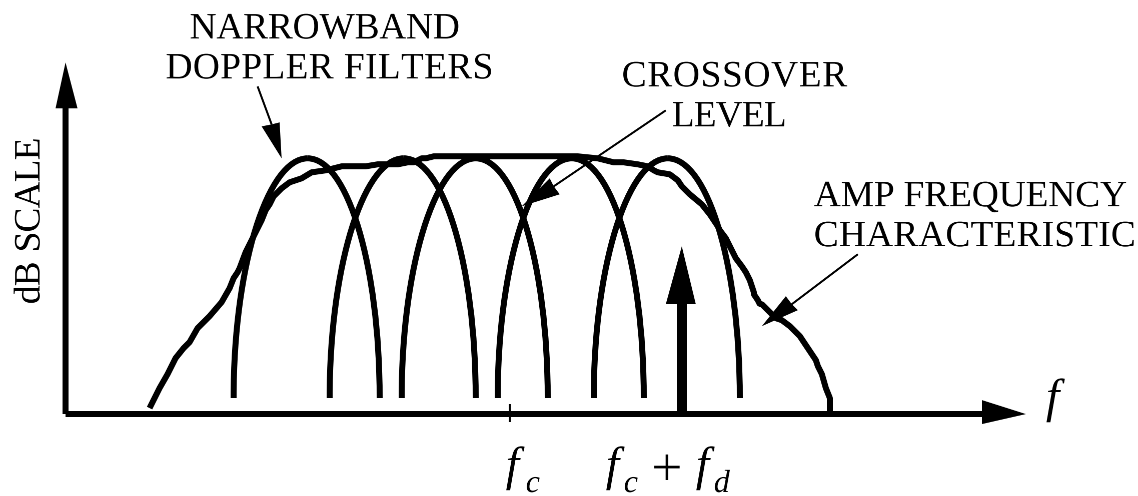
Another problem: the f_d signal is narrow and the amplifier B_n much larger so the SNR is too low.



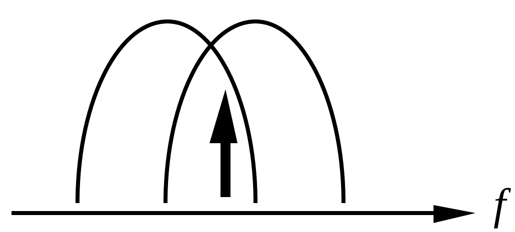
The solution is to use a collection of filters with narrower bandwidths.

Doppler Filter Banks

The radar band is divided into narrow sub-bands. Ideally there should be no overlap in their frequency characteristics.



The noise bandwidth of the doppler filters is small compared to that of the amplifier, which improves the SNR. Velocity estimates can be made by monitoring the power out of each filter. If a signal is present in a filter, the target's velocity range is known. The energy outputs from adjacent filters can be used to interpolate velocity.



Example

CW radar example:

$f = 18$ GHz (l = 0.0167 m)
target speed range: mach 0.25 to mach 3.75 (mach 1 = 334.4 m/s)
resolution = 10 m/s
homodyne receiver

(a) What is the required spectral range for the filter bank?

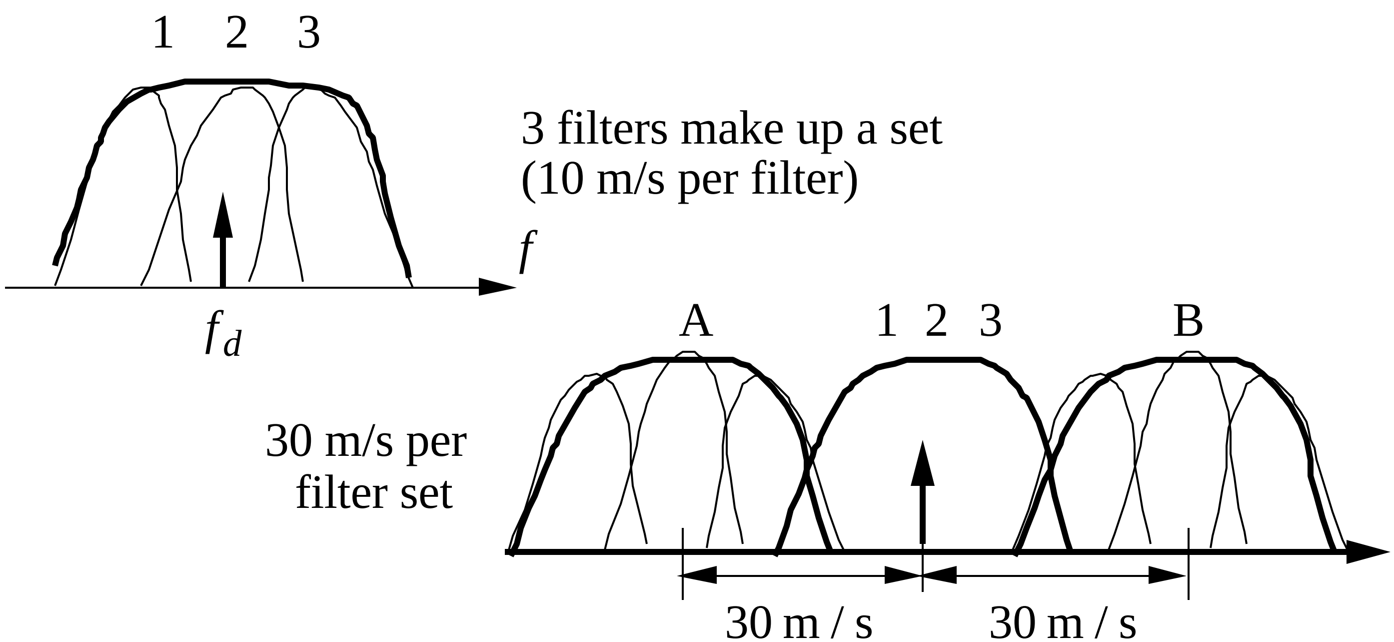
$$\left.\begin{array}{l} \text{lowest frequency}: f_{d1} = \dfrac{(2)(83.6)}{0.0167} = 10\,\text{kHz} \\ \text{highest frequency}: f_{d2} = \dfrac{(2)(1254)}{0.0167} = 150\,\text{kHz} \end{array}\right\} \Rightarrow 10\,\text{kHz} \leq f \leq 150\,\text{kHz}$$

(b) How many filters are required?

$$N_f = \frac{\Delta f_2 - \Delta f_1}{\Delta f} = \frac{v_{r2} - v_{r1}}{\Delta v} = \frac{1254 - 83.6}{10} = 117$$

Example

(c) When a target is located in a particular filter, the two adjacent filters are also monitored in case the velocity changes. It takes 0.5 second to shift between filter sets. How fast must a target accelerate to defeat the radar?



To defeat the radar the target's doppler must jump to A or B

$$a_r = (\pm 30 \text{ m/s}) / 0.5 \text{ s} = \pm 60 \text{ m/s}^2$$

I and Q Representation

Coherent detection requires dealing with the envelope of a signal, $g(t)$ and the phase of the sinusoidal carrier, $\Phi(t)$. They need not be measured directly, but can be derived using in-phase (I) and quadrature (Q) channels as follows. For narrowband signals we can write

$$s(t) = g(t)\cos\big(w_c t + \Phi(t)\big)$$

or, in terms of I and Q components

$$s(t) = g_I(t)\cos(w_c t) - g_Q(t)\sin(w_c t)$$

where

$$g_I(t) = g(t)\cos(\Phi(t))$$

$$g_Q(t) = g(t)\sin(\Phi(t))$$

Define the complex envelope of the signal as

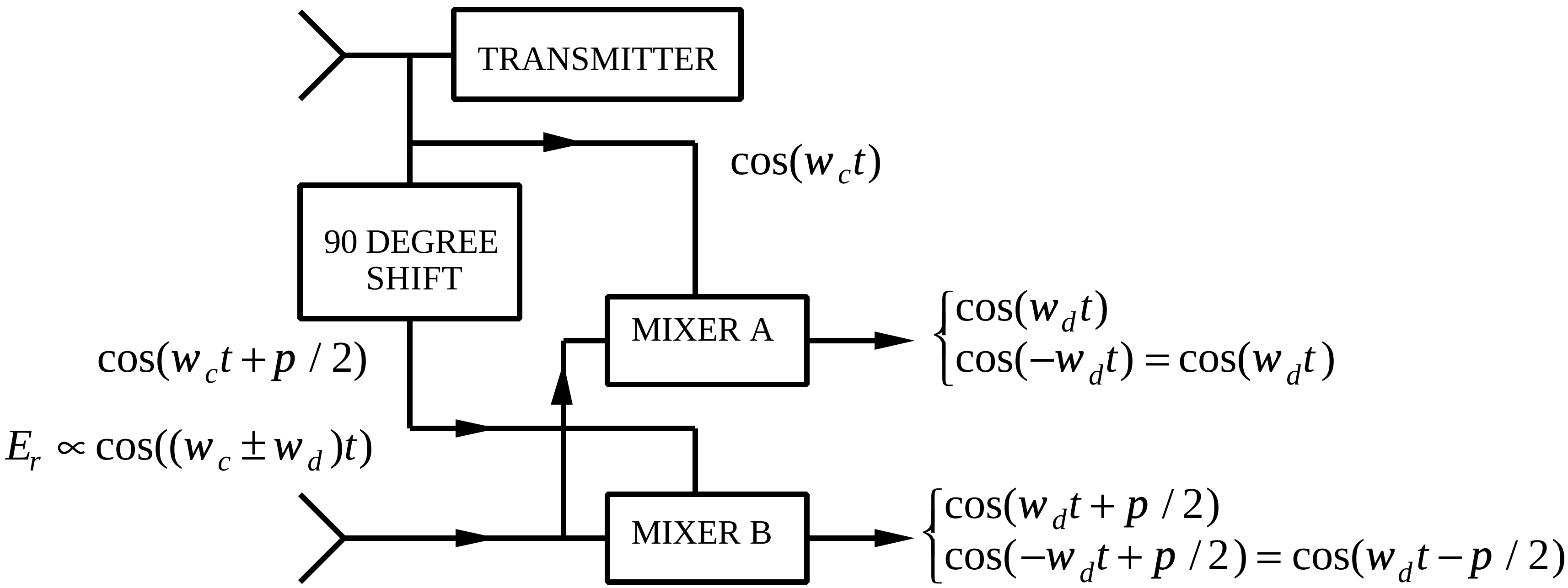
$$u(t) = g_I(t) + j\, g_Q(t)$$

Thus the narrowband signal can be expressed as

$$s(t) = \mathrm{Re}\left\{u(t)e^{j w_c t}\right\}$$

Doppler Frequency Shift (4)

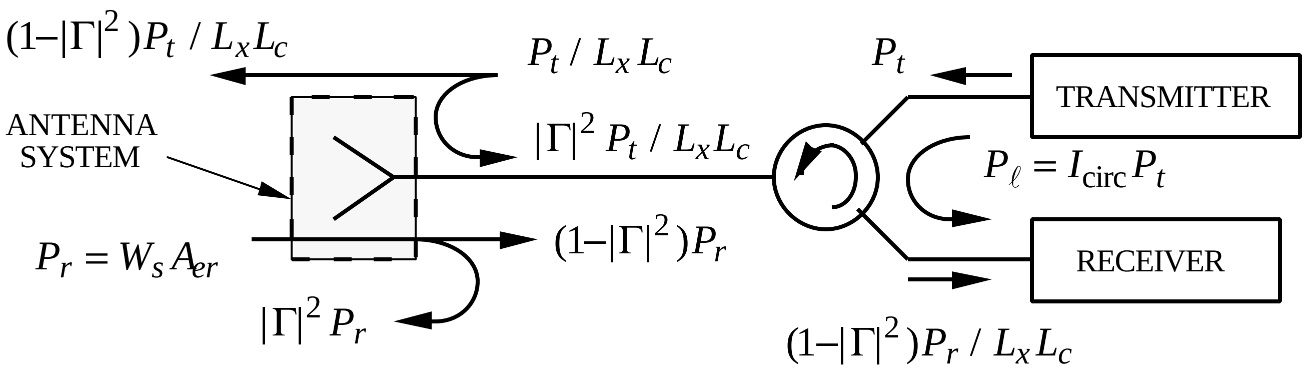
Recovering the sign of the doppler shift using *I* and *Q* channels



Positive doppler shift results in a phase lead; negative doppler shift results in a phase lag.

CW Radar Problems (1)

1. <u>Transmit/receive leakage</u>:



P_t = power out of transmitter
P_ℓ = power leaked directly from transmitter to receiver
P_r = power scattered by target and collected by the antenna
$|\Gamma|$ = magnitude of the antenna voltage reflection coefficient (related to its VSWR)
L_x = transmission line loss (≥ 1)
L_c = circulator loss (≥ 1)
$I_{\rm circ}$ = circulator isolation (fraction of incident power leaked in the reverse direction)

If higher order reflections can be ignored, then the total signal at the receiver is

$$P_{\rm tot} = \underbrace{(1-|\Gamma|^2)P_r\ /\ L_x L_c}_{\text{TARGET RETURN}} + \underbrace{I_{\rm circ} P_t}_{\text{LEAKAGE}} + \underbrace{|\Gamma|^2\ P_t\ /(L_x L_c)^2}_{\text{ANTENNA MISMATCH}}$$

CW Radar Problems (2)

Comments regarding leakage:

1. Circulator isolation is in the range of 30 to 100 dB. It can be increased at the expense of size, weight, volume, and insertion loss.
2. Leakage can be reduced by using two separate antennas. There is still leakage which arises from
   - -- near field coupling of the antennas
   - -- reflection from close in clutter
   - -- surface guided waves on platform or ground

Example: receiver MDS = -130 dBW (= -100 dBm)
peak transmitter power = 100 W = 20 dBW

To keep the leakage signal below the MDS requires

$$P_\ell = I_{\mathrm{circ}} P_t \le -130\,\mathrm{dBW} \underset{\text{IN dB}}{\Rightarrow} 20\,\mathrm{dBW} - I_{\mathrm{circ}},\ \mathrm{dB} \le -130\ \mathrm{dBW} \Rightarrow -I_{\mathrm{circ}},\ \mathrm{dB} = -150\ \mathrm{dB}$$

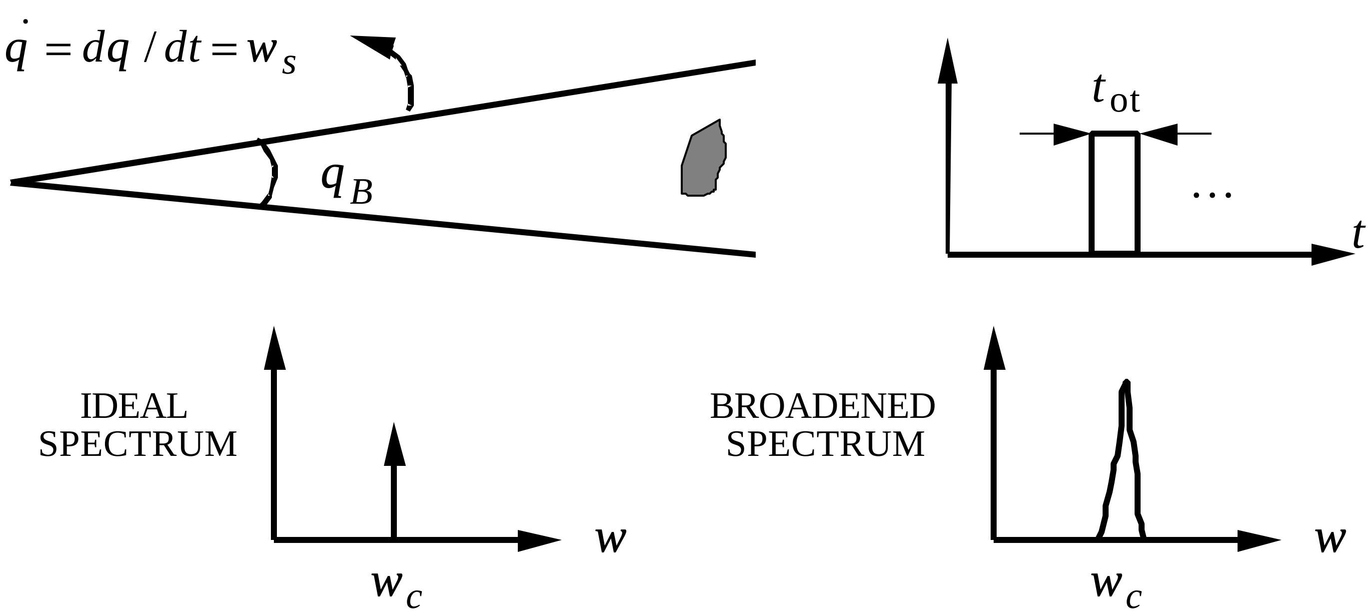
Note that circulator isolation is given in positive dB, but the negative sign is implied. Thus the required isolation is

$$I_{\mathrm{circ}},\ \mathrm{dB} = 150\ \mathrm{dB}$$

CW Radar Problems (3)

2. Spectrum broadening caused by:

   1. Finite duration illumination due to antenna scanning

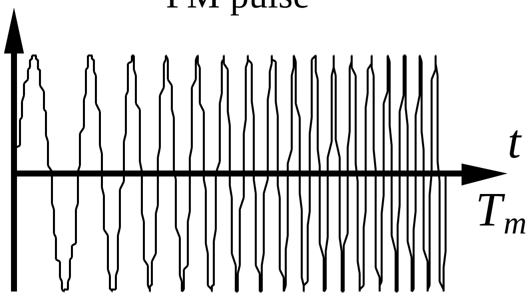


   2. Modulation of the echo by target moving parts and aspect changes
   3. Acceleration of the target

Frequency Modulated CW (FMCW)

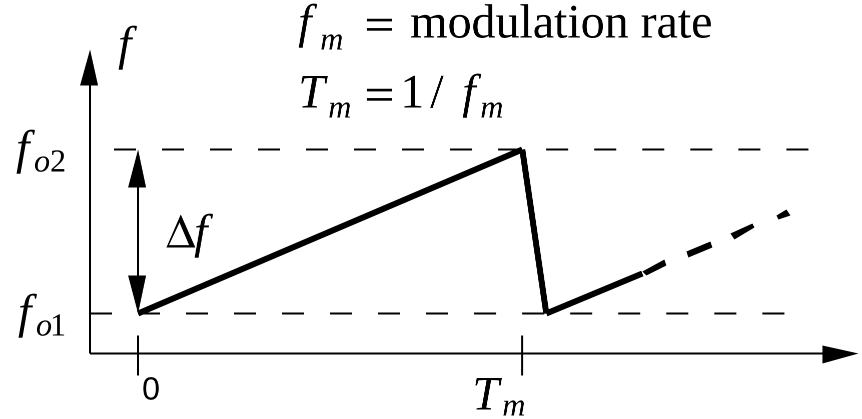
Conventional CW radars cannot measure range. To do so the transmit waveform must be "tagged." This can be done by modulating the frequency periodically with time. This technique is called <u>FM ranging</u>.

Typical linear FM pulse

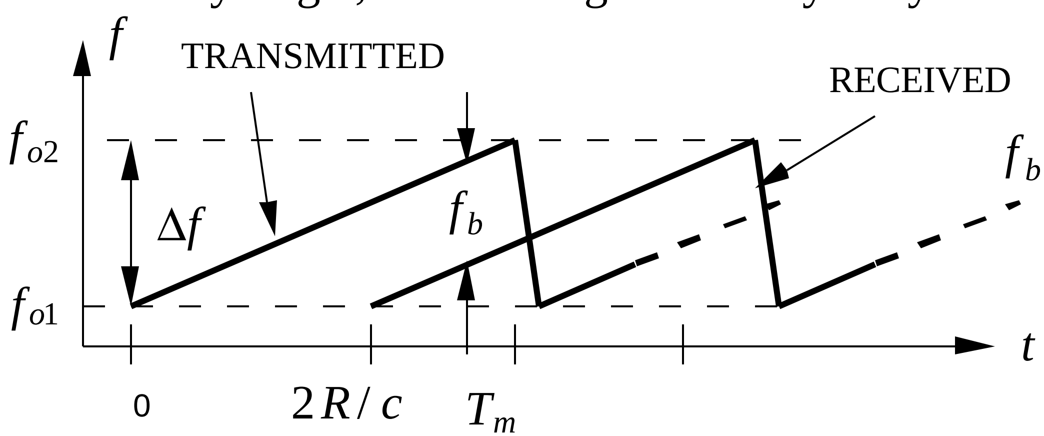


f_m = modulation rate

$T_m = 1/f_m$



For a stationary target, the echo signal is delayed by the transit time $2R/c$

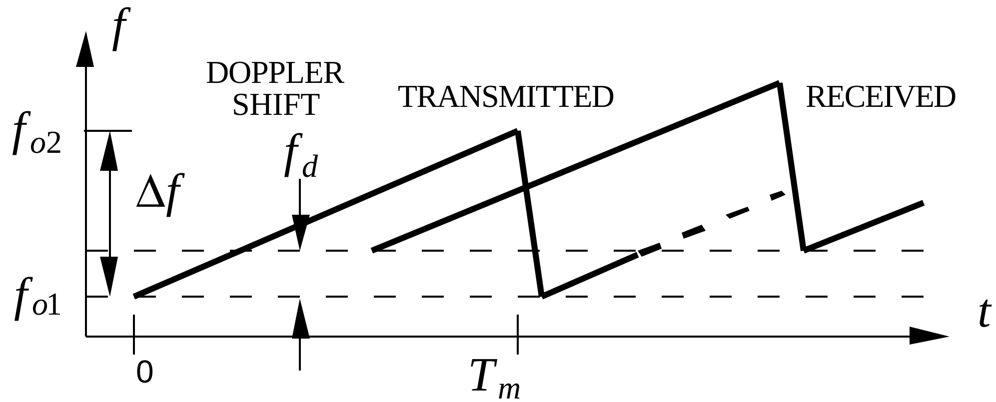


f_b = beat frequency ($= f_r$)

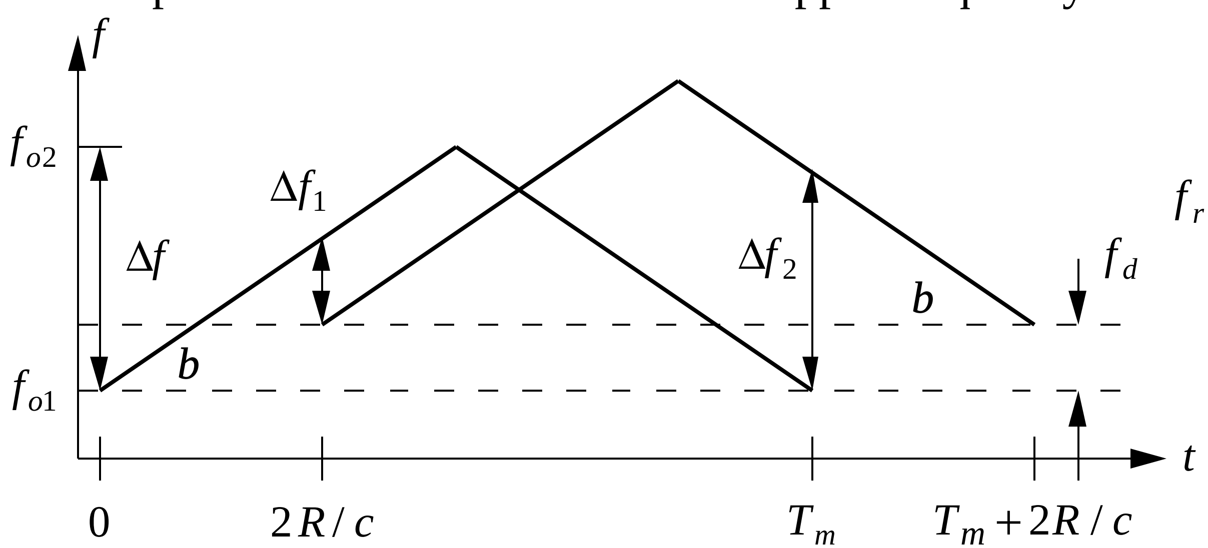
$$= \frac{2R}{c}\frac{\Delta f}{T_m}$$

FMCW (2)

If the target is in motion the received waveform might be doppler shifted



Use a two-slope modulation to eliminate the dopper frequency



$$f_r = \frac{2R}{c}\frac{\Delta f}{T_m/2}$$
$$= \frac{4R}{c}\Delta f \;\; f_m$$

FMCW (3)

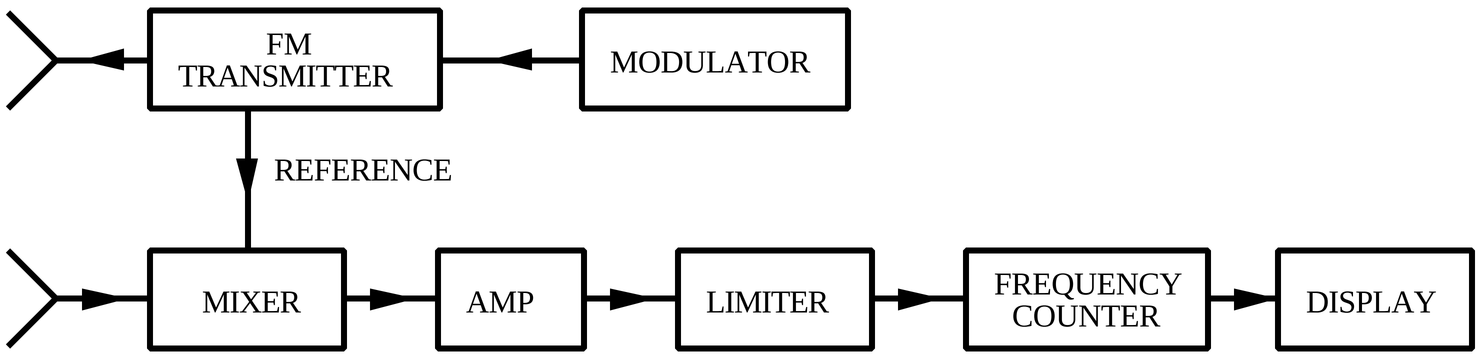
Range can be measured if Δf_1 and Δf_2 are known

$$\tan b = (\Delta f_1 + f_d)/(2R/c)$$
$$\tan b = (\Delta f_2 - f_d)/(2R/c)$$

Add the two equations

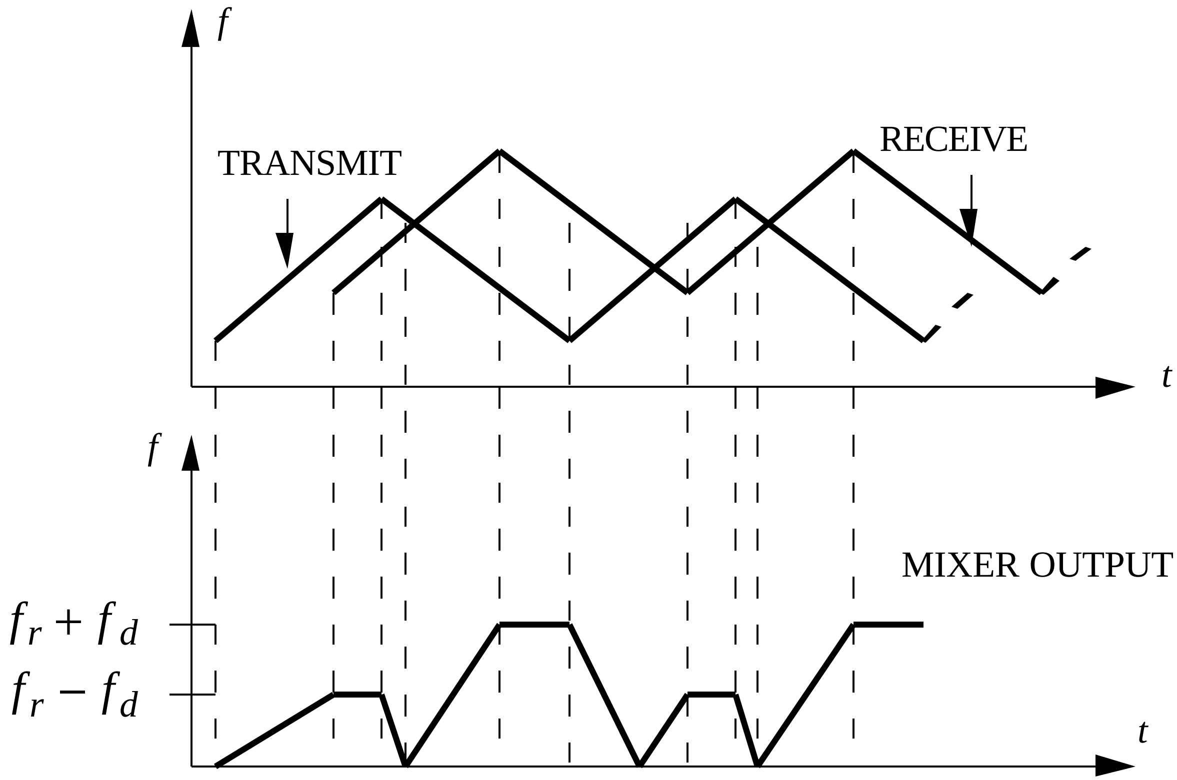
$$2\tan b = (\Delta f_1 + \Delta f_2)/(2R/c) \Rightarrow R = \frac{(\Delta f_1 + \Delta f_2)c}{4\tan b}$$

Block diagram:



FMCW (4)

Waveform out of mixer:

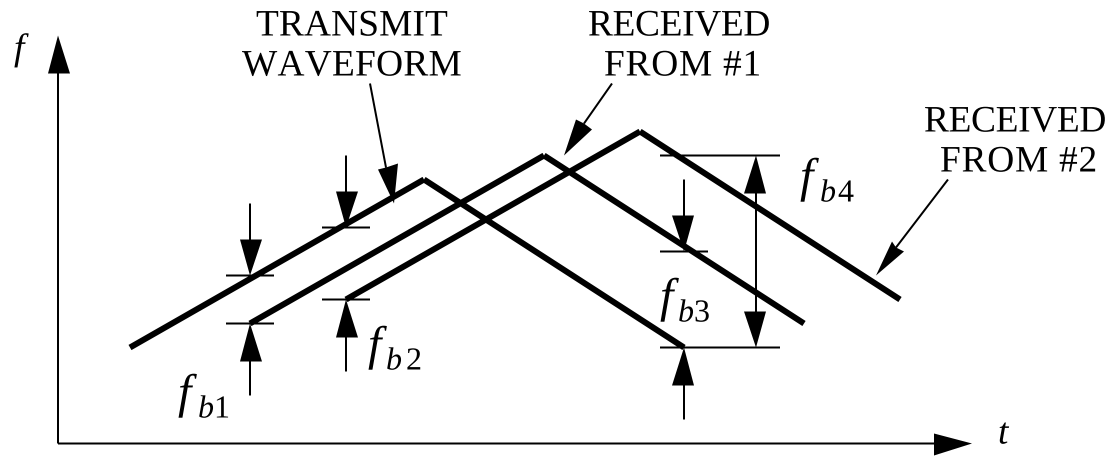


Restriction:

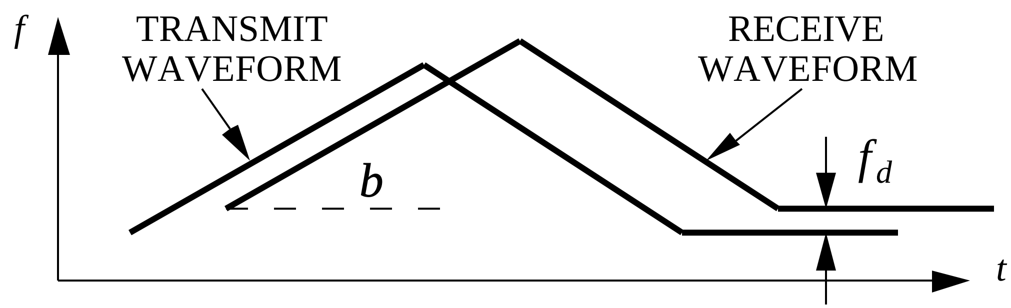
$$\frac{1}{f_m} >> \frac{2R}{c}$$

FMCW Complications

1. Multiple targets cause "ghosting." There are two beat frequencies at the end of each segment. The radar does not know how to pair them.

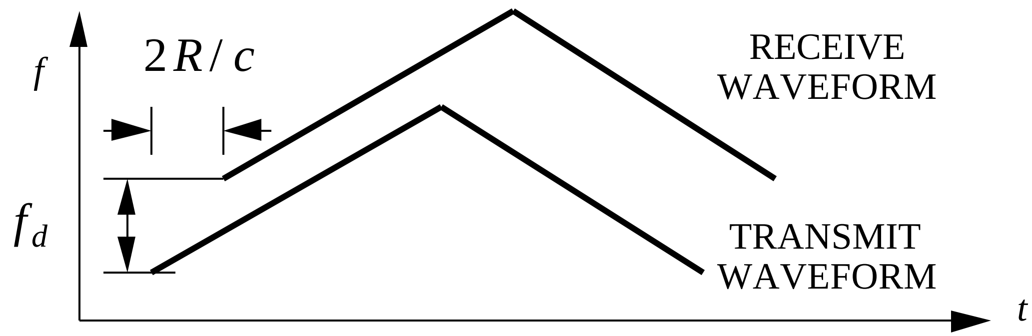


Solution: add a constant frequency segment to measure f_{d1} and f_{d2}



FMCW Complications

2. Doppler frequency greater than $\frac{2R}{c}\tan b$ (which occurs in air-to-air situations) complicates the formulas for the frequency differences.



Sinusoidal modulation can also be used. The range is determined from the average (over one cycle) beat frequency.

Example: Find the range if the low and high beat frequencies out of the mixer are $\Delta f_1 = f_r - f_d = 4825$ Hz and $\Delta f_2 = f_r + f_d = 15175$ Hz and the sweep rate is $\tan b = 10$ Hz/$m$sec $= 10 \times 10^6$ Hz/sec.

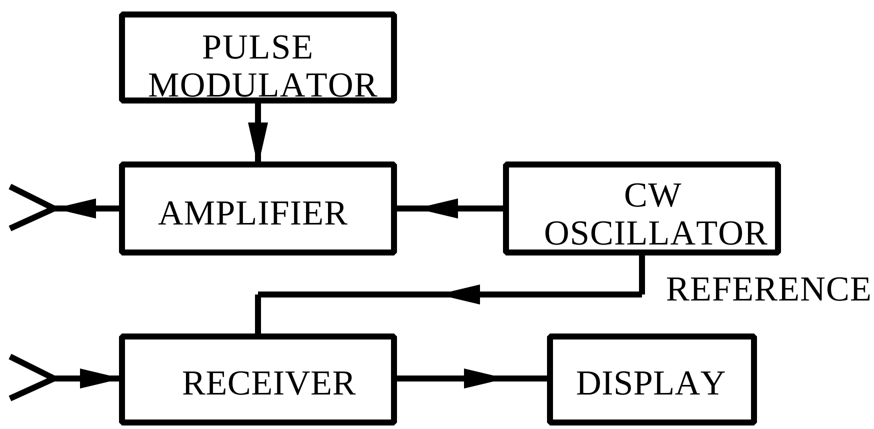
Compute range : $R = \dfrac{(15175 + 4825)(3 \times 10^8)}{4(10 \times 10^6)} = 150000\,\text{m}$

MTI and Pulse Doppler Radar

The doppler frequency shift can be used with pulse waveforms to measure velocity. The radars generally fall into one of two categories:

1. moving target indication (MTI)
   - generally uses delay line cancelers
   - ambiguous velocity measurement
   - unambiguous range measurement
2. pulse doppler radar
   - generally uses range gated doppler filters
   - unambiguous velocity measurement
   - ambiguous range measurement

The main feature of both is the use of a coherent reference signal.

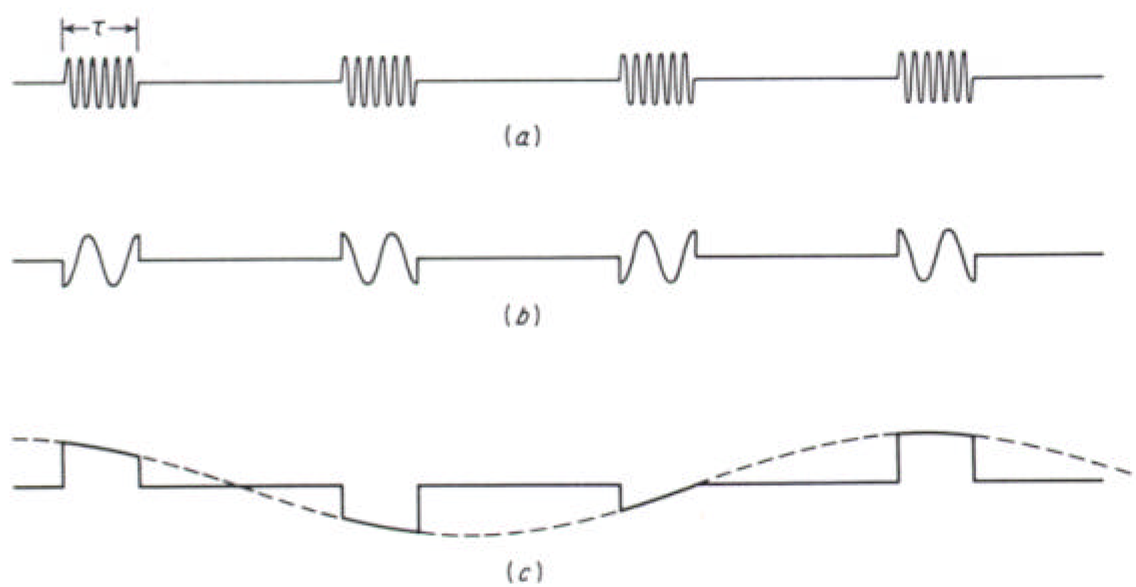


MTI (1)

The doppler is a sinusoidal modulation of the transmitted waveform. For a pulse train:

1. large doppler shift: modulation of the waveform is rapid (e.g., a ballistic missile). Only need one or two pulses to measure doppler shift
2. small doppler shift: modulation of the waveform is slow (e.g., aircraft). Need many pulses to measure doppler shift

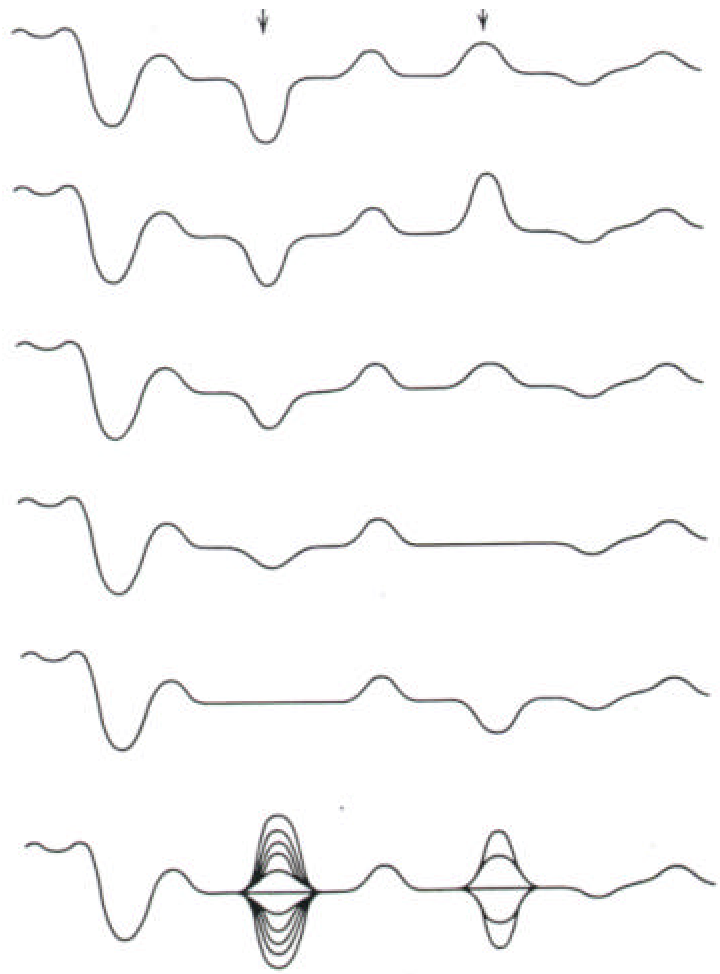
("Large" and "small" are by comparison to $1/t$.) From Fig. 3.4 in Skolnik: (a) RF echo pulse train, (b) video pulse train for $f_d > 1/t$, (c) for $f_d < 1/t$.



MTI (2)

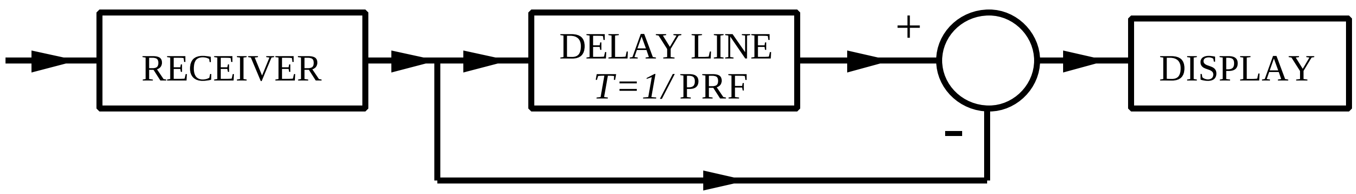
On successive "A" scope traces (amplitude vs. range), moving target returns will vary in amplitude; fixed target returns are constant.

Fig. 3.5 in Skolnik (the bottom curve is the superposition of many sweeps)



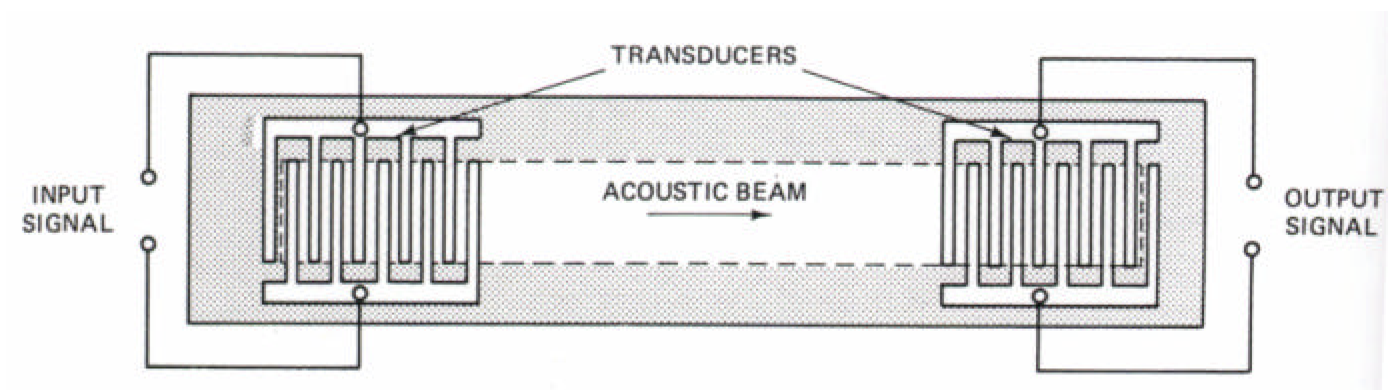
MTI (3)

PPI displays use a delay line canceler



If two successive pulses are identical then the subtracted signal is zero; if two successive pulses are not identical then the subtraction results in a nonzero "residue."

The delay time is equal to the PRF. Typical values are several milliseconds, which requires very long line lengths for electromagnetic waves. Usually acoustic devices are used. The velocity of acoustic waves is about 10^{-5} of that for electromagnetic waves.



(From *Acoustic Waves: Devices, Imaging andAnalog Signal Processing*, by Kino)

PD and MTI Problem: Eclipsing

To protect the receiver from transmitter leakage, the receiver is usually shut down during the transmission of a pulse. Eclipsing occurs when a target echo arrives during a transmit segment, when the radar receiver is shut down.

Points to note:

1. Targets are generally not completely eclipsed; usually some of the return gets through.
2. Partial eclipsing results in a loss of SNR because some of the target return is discarded.
3. The average eclipsing loss is given approximately by

$$L_{ec} \approx \int_{0}^{t} (t/t)^2 \, dt + \int_{t}^{t_v} dt + \int_{t_v}^{t_v+t} (t_v + t - t)^2 / t \,)dt$$

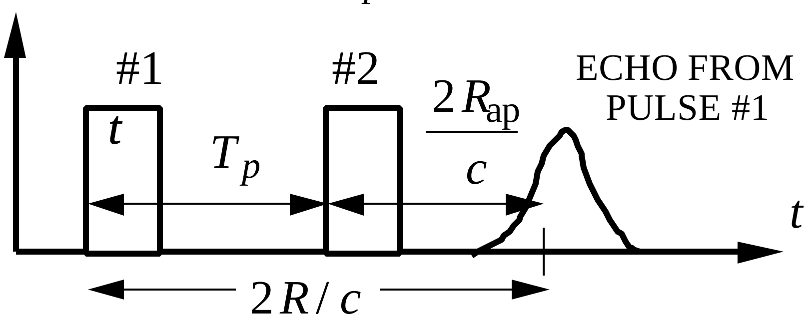
   where t_v is the video integration time. Normalization by the PRF gives

$$L_{ec} \approx (\underbrace{t_v \cdot \mathrm{PRF}}_{\substack{\text{RECEIVE} \\ \text{DUTY FACTOR}}} - \underbrace{t \cdot \mathrm{PRF/3}}_{\substack{\text{TRANSMIT} \\ \text{DUTY FACTOR}}})$$

4. Eclipsing can be reduced by switching PRFs.

PD and MTI Problem: Range Ambiguities

Maximum unambiguous range is $R_u = cT_p/2$. A low PRF is desirable to maximize R_u.



Example: PRF $= 800\,\text{Hz}$, $T_p = 0.00125\text{sec}$, $R = 130$ nmi (nautical miles)

- Based on the true range

$$\frac{2R}{c} = \frac{2(130)}{\underbrace{161875}_{c \text{ IN nmi/sec}}} = 0.001606 = 1.606 \text{ ms}$$

- Based on the apparent range

$$\frac{2R_{ap}}{c} = \frac{2R}{c} - T_p = 0.001606 - 0.00125 = 0.356 \text{ ms} \Rightarrow R_{ap} = 28.8 \text{ nmi}$$

Range Ambiguities (2)

The apparent range depends on the PRF but the true range does not. Therefore, changing the PRF can be used to determine whether the range is true or apparent. This is called PRF switching, pulse staggering or multiple PRFs.

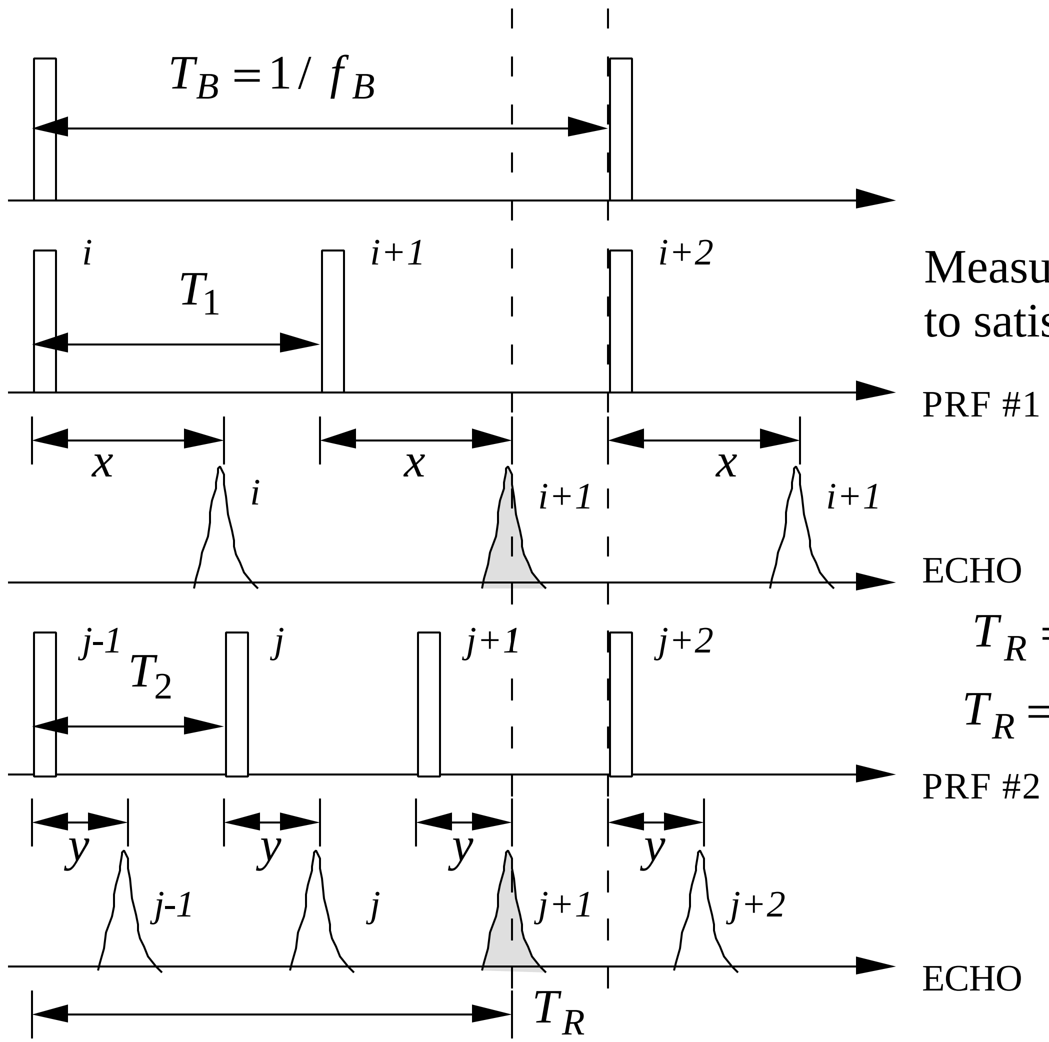
Choose two PRFs: $f_1 = N_1 f_B$ and $f_2 = N_2 f_B$ where

f_B is the basic PRF (usually set by the unambiguous range)
N_1, N_2 are relatively prime integers (e.g., 11 and 13)

PRF switching method:

1. Transmit two PRFs and look for a common return which signifies a true range
2. Count the number of elapsed pulses to get the integers i and j
3. Measure x and y
4. Compute T_R and then R

Range Ambiguities (3)



Measure x, y and choose i and j to satisfy both equations:

$$T_R = (i+1-1)T_1 + x = iT_1 + x$$

$$T_R = (j+1-1)T_2 + y = jT_2 + y$$

Example

We want $R_u = 100$ nmi or $f_B = \dfrac{c}{2R_u} = \dfrac{161875}{2(100)} = 810\ \text{Hz}$

Choose $N_1 = 79,\ N_2 = 80$:

$$f_1 = N_1 f_B = 79(810) = 63.990\ \text{kHz}$$

$$f_2 = N_2 f_B = 80(810) = 64.800\ \text{kHz}$$

Unambiguous ranges:

$$R_{u1} = c/(2f_1) = \frac{161875}{2(63.990)10^3} = 1.2648\ \text{nmi}$$

$$R_{u2} = c/(2f_2) = \frac{161875}{2(64.800)10^3} = 1.2490\ \text{nmi}$$

Assume that the target is at 53 nmi

$$T_R = \frac{2(53)}{161875} = 654.8 \times 10^{-6}\ \text{sec}$$

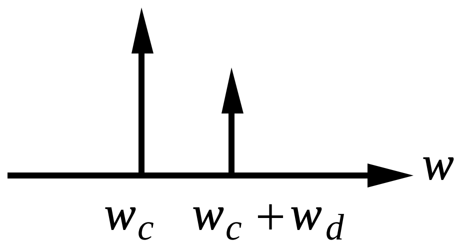
Subtract out integers

$$T_R f_1 = i + x\, f_1 = 41.902 \quad \Rightarrow \quad x = 0.902\, T_1 = 14.096\, m\text{sec}$$

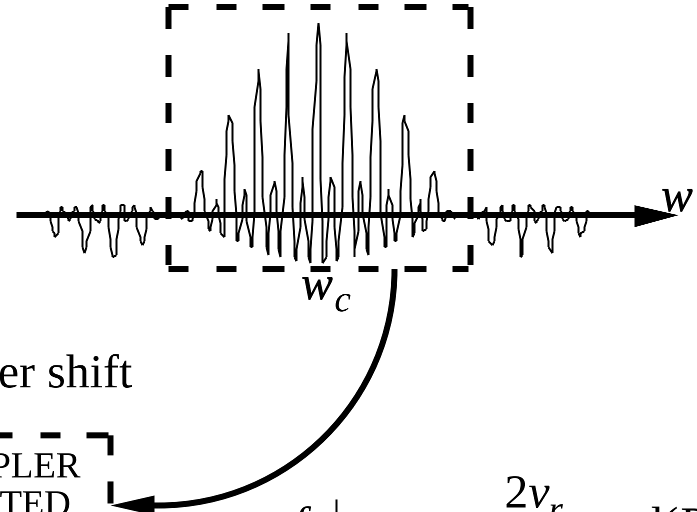
$$T_R f_2 = j + y\, f_2 = 42.432 \quad \Rightarrow \quad y = 0.432\, T_2 = 6.67\, m\text{sec}$$

PD and MTI Problem: Velocity Ambiguities

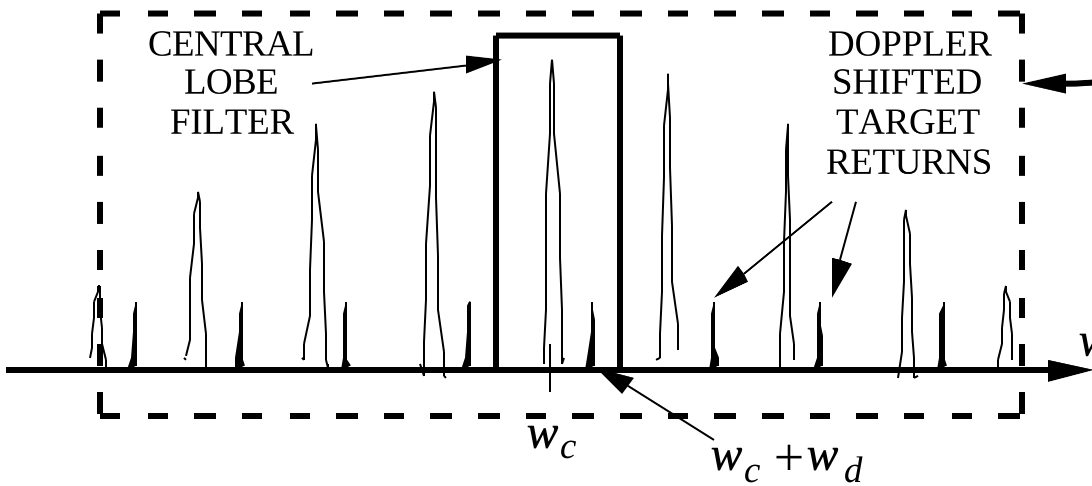
Spectrum of doppler shifted CW signal



Coherent pulse train spectrum (fixed target -- no doppler)



Expanded central lobe region with target doppler shift

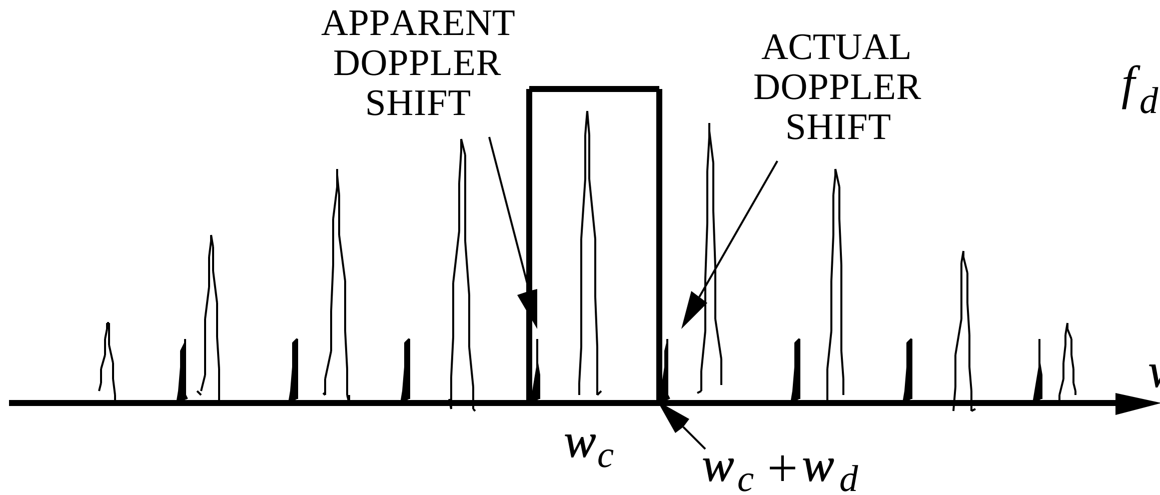


$$f_d\big|_{\text{observed}} = \frac{2v_r}{l}\ \text{mod(PRF)}$$

$$f_d = n\,\text{PRF} + f_d\big|_{\text{apparent}}$$

Velocity Ambiguities (2)

If ω_d is increased the true target doppler shifted return moves out of the passband and a lower sideband lobe enters. Thus the doppler measurement is ambiguous.



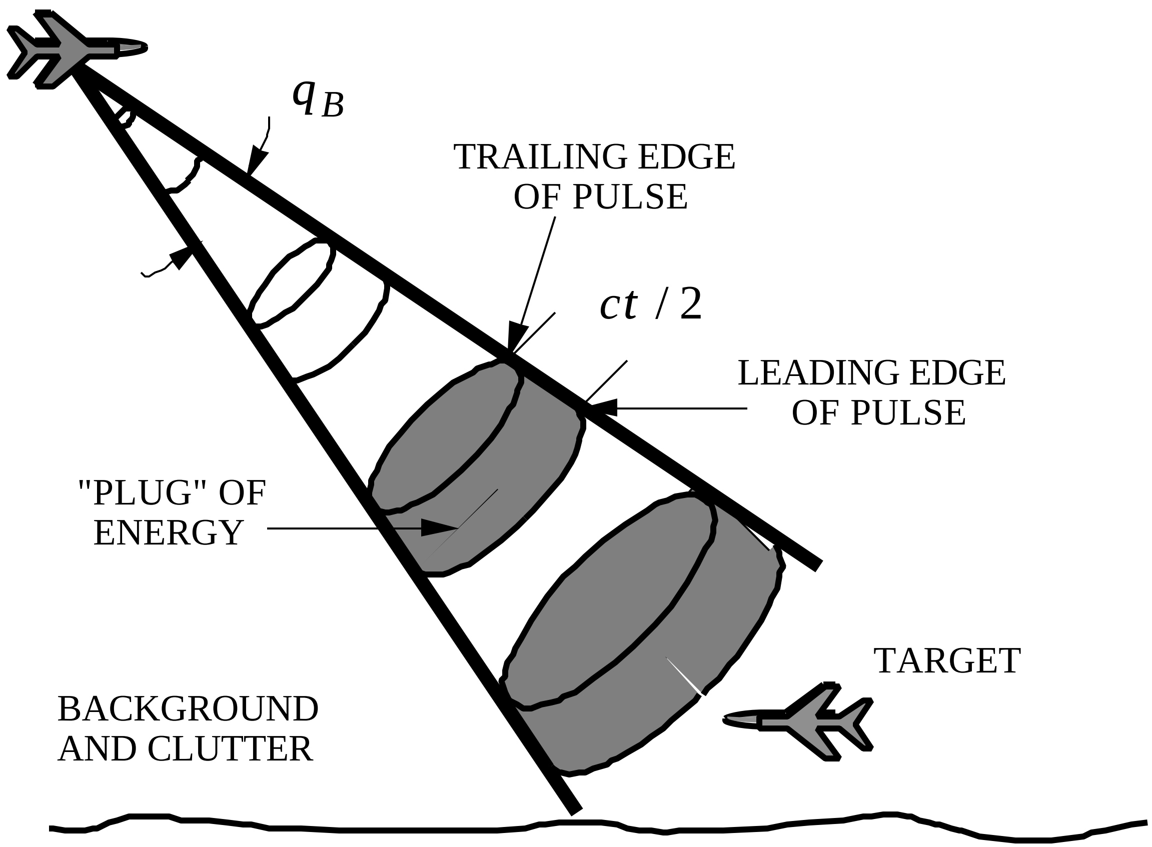
$$f_{d\,\max} = \pm f_p / 2$$
$$v_u = \lambda\ f_{d\,\max} / 2$$
$$= \pm \lambda\ f_p / 4$$
$$\Delta v_u = \lambda\ f_p / 2$$

PRF determines doppler and range ambiguities:

PRF	RANGE	DOPPLER
High	Ambiguous	Unambiguous
Medium	Ambiguous	Ambiguous
Low	Unambiguous	Ambiguous

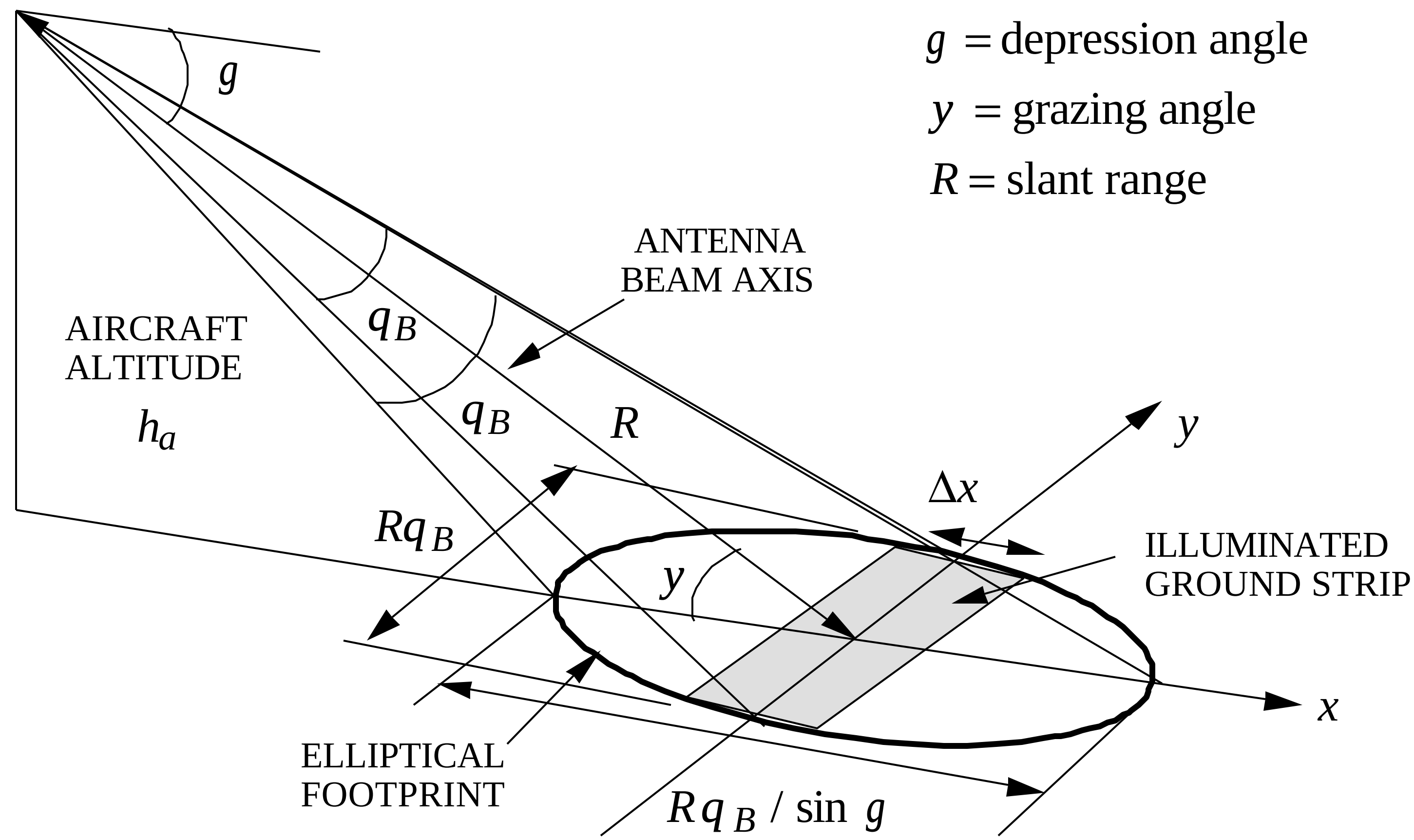
Airborne MTI and Pulse Doppler Operation

Airborne MTI (AMTI) refers to any MTI operating on a moving platform. Motion effects include clutter spectrum frequency shift and broadening.



Surface Clutter (1)

Airborne radar illuminates the ground. Simple model for the <u>footprint</u> dimensions:



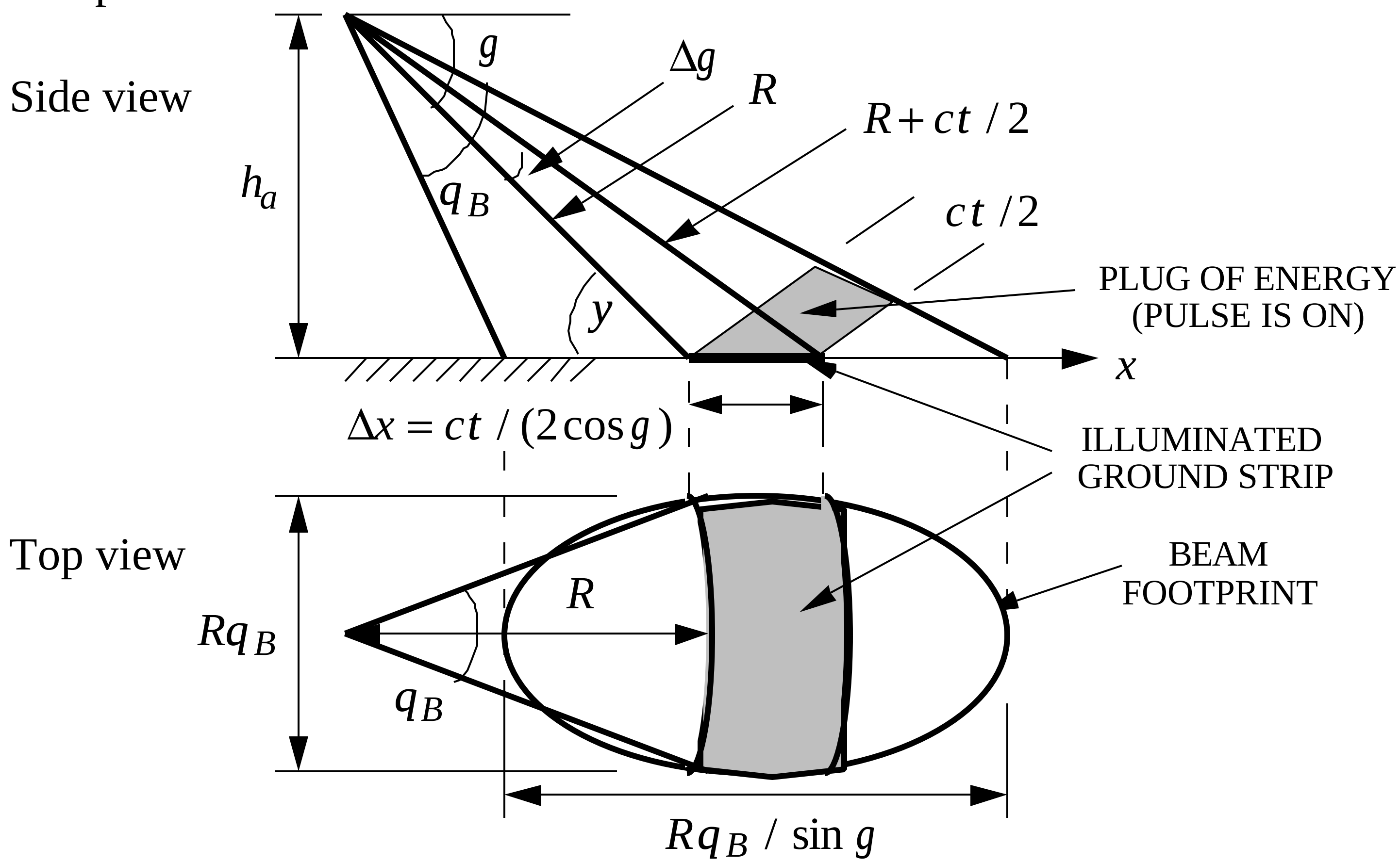
g = depression angle

y = grazing angle

R = slant range

Surface Clutter (2)

The illumination strip is actually curved but is approximately rectangular if R is large and q_B is small. The diagram illustrates the <u>pulse width limited</u> case; only a portion of the footprint is illuminated.



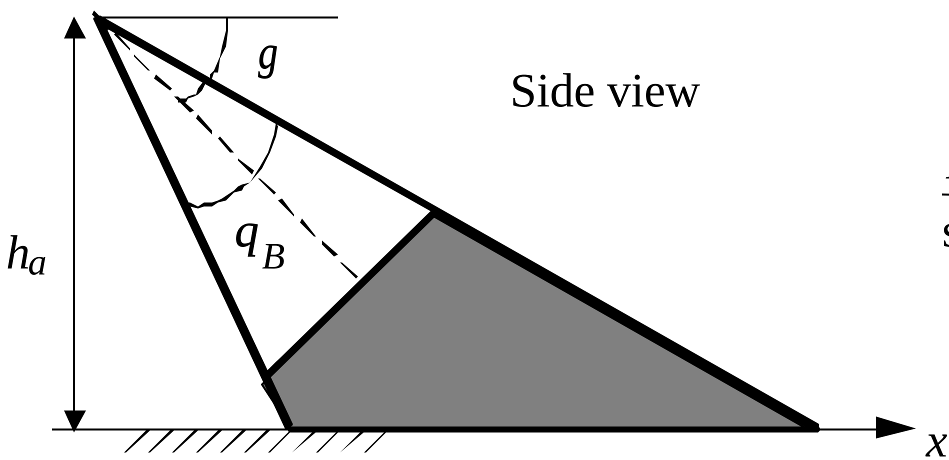
Surface Clutter (3)

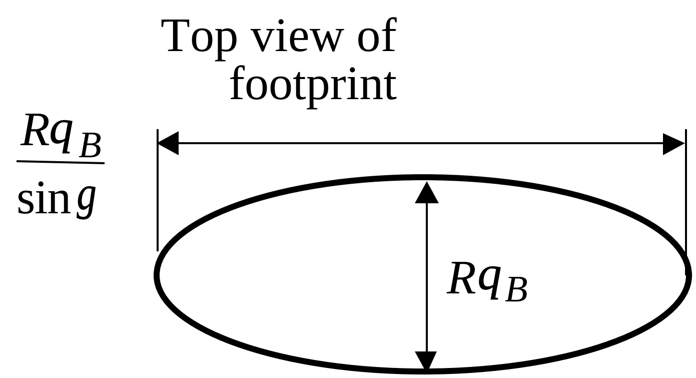
For a small depression angle the clutter area is approximately given by

$$A_c = \frac{c\tau\, R\theta_B}{2\cos\gamma}$$

The diagram below illustrates the <u>beamwidth limited</u> case; the entire footprint is illuminated. The clutter area is the area of an ellipse

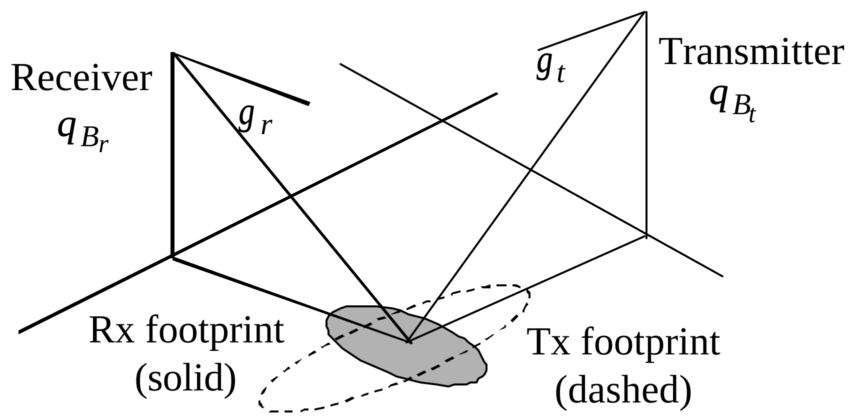
$$A_c = \frac{\pi}{4}(R\theta_B)(R\theta_B/\sin\gamma) = \frac{\pi(R\theta_B)^2}{4\sin\gamma}$$





Two-Way Pattern Beamwidth

The <u>two-way pattern</u> refers to the product $G_t G_r$, and the half power points of this product define the two-way beamwidths. The previous charts have specifically dealt with mono-static radar. In a more general (bistatic) case, the clutter area is determined by the intersection of the transmit and receive footprints. The figure shows a bistatic radar with different transmit and receive patterns. The clutter area is determined by the overlap in the two beams, or equivalently, for a monostatic radar, the two-way beamwidth. For a gaussian beam of the form



$$G(q) = G_o \exp\left\{-(2.776)(q/q_B)^2\right\},$$

the two-way pattern is

$$G(q)^2 = G_o^2 \exp\left\{-(2)(2.776)(q/q_B)^2\right\},$$ and thus $q_{B_{2-\text{way}}} = \dfrac{q_{B_{1-\text{way}}}}{\sqrt{2}}$. Therefore, the clutter area equations should have an additional factor of 2 in the denominator ($\sqrt{2}$ for each plane), because A_c should be based on the two-way beamwidths. However, the area is usually greater than that found using the additional ½ because the beam edges are not sharp, so area outside of the 3 dB beamwidth contributes.

Surface Clutter (4)

For extended targets we define the cross section per unit area, s^o, in m^2/m^2

$$s^o = \underbrace{\text{Area}}_{1\text{ m}^2} \times \underbrace{\text{Reflectivity}}_{\Gamma_g} \times \underbrace{\text{Directivity}}_{F_g}$$

This quantity is tabulated for various surfaces (for example, see Fig. 7.3 in Skolnik, reproduced on page II-63).

The radar equation for clutter return is

$$C = \frac{P_t G_t A_e s^o A_c}{(4p)^2 R^4}$$

Neglecting noise ($C >> N$), the signal-to-clutter ratio (SCR) is

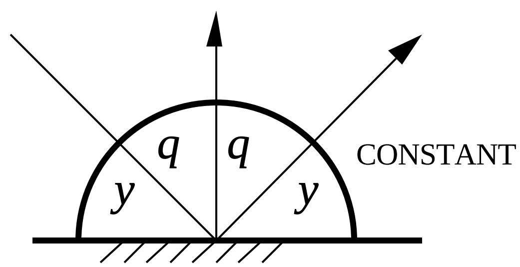
$$\text{SCR} = \frac{S}{C} = \frac{s}{s^o A_c}$$

where s is the RCS of the target. Note: (1) P_t does not affect the SCR, and (2) a large t decreases N, but increases C

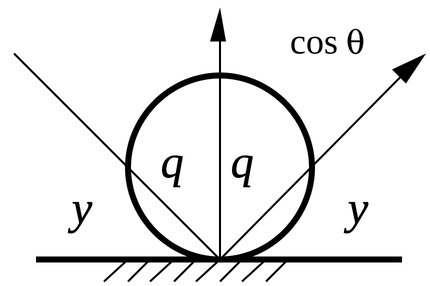
Backscatter From Extended Surfaces

Extended surfaces: have no edges that are illuminated

ISOTROPIC (DIFFUSE) SURFACE



LAMBERTIAN SURFACE



In general:

$$G_g = \frac{2}{\int_0^{p/2} F_g(q)\sin q \, dq}$$

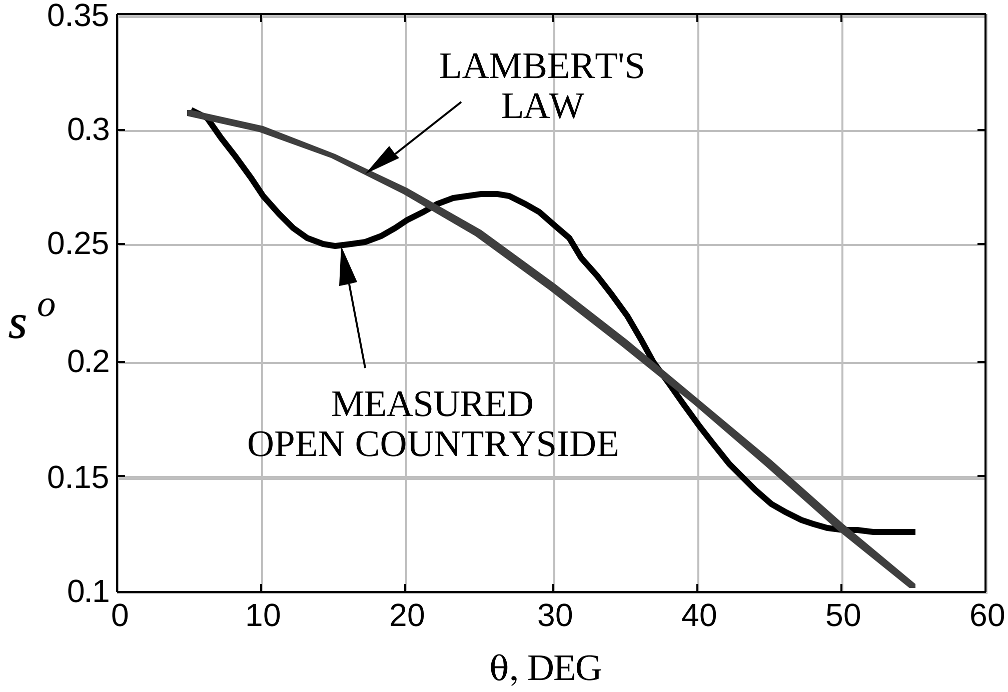
<u>Isotropic surface</u>: $F_g(q) = 1 \Rightarrow G_g = \frac{2}{\int_0^{p/2} \sin q \, dq} = 2 \Rightarrow s^o(q) = 2\Gamma_g \cos q$

<u>Lambertian surface</u>:

$$F_g(q) = \cos q \Rightarrow G_g = \frac{2}{\int_0^{p/2} \cos q \sin q \, dq} = 4 \Rightarrow s^o(q) = 4\Gamma_g \cos^2 q$$

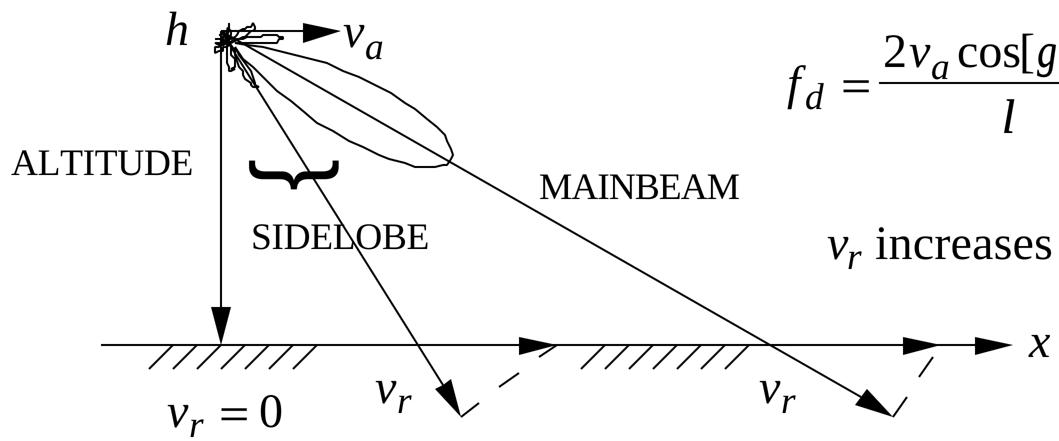
Backscatter From Extended Surfaces

- HH polarization, L-band



Clutter Spectrum (1)

Each point on the surface has a different velocity relative to the platform:



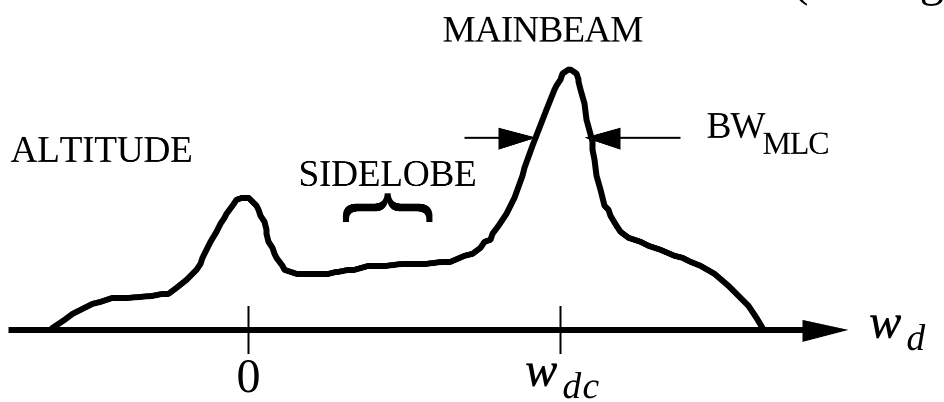
$$f_d = \frac{2 v_a \cos[g\,(x)]}{l}$$

Three components of surface return:

1. <u>mainbeam</u> clutter: high because of high antenna gain
2. <u>sidelobe</u> clutter: low but covers many frequencies because of the large angular extent of the sidelobes
3. <u>altitude</u> return: high because of normal incidence

Clutter Spectrum (2)

Typical clutter spectrum: (see Fig. 3.43 in Skolnik)



Mainbeam clutter characteristics:

1. as the half-power beamwidth (HPBW) increases the spread in v_r increases and hence w_d increases
2. as g increases v_r at the center of the footprint decreases and therefore w_{dc} decreases
3. the width and center of frequencies varies as 1/ l

$$\frac{df_d}{dg} = \frac{d}{dg}\left(\frac{2v_a}{l}\cos g\right) = -\frac{2v_a}{l}\sin g \equiv \frac{\Delta f_d}{\Delta g}$$

Clutter Spectrum (3)

Mainbeam clutter characteristics (continued):

$$\Delta f_d = \mathrm{BW}_{\mathrm{MLC}} \approx \frac{2v_a}{l}\sin g\ \Delta g$$

where Δg is the antenna beamwidth between first nulls. For a circular aperture of radius a, the 3-dB and first null beamwidths are $29.2\, l\,/\,a$ and $69.9\, l\,/\,a$, respectively. Therefore use the approximation $\Delta g \approx 2.5 q_B$ so that

$$\Delta f_d = \mathrm{BW}_{\mathrm{MLC}} \approx \frac{2v_a}{l}\sin g\ (2.5 q_B)$$

4. Azimuth scanning: forward: high, narrow spectrum
   broadside: low, broad spectrum

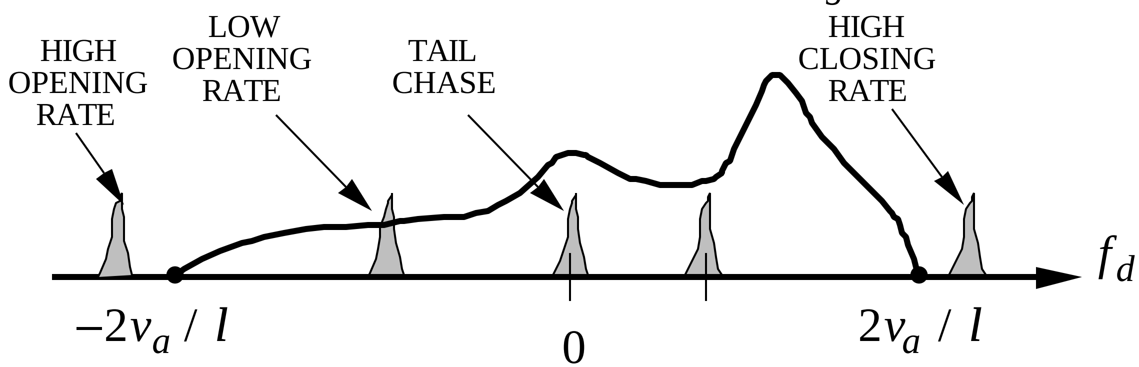
Sidelobe clutter characteristics:

1. lowering the sidelobes reduces the clutter
2. depends on terrain Γ_g and F_g
3. most severe at moderate altitudes
4. in practice, extends from $-\frac{2v_a}{l}$ to $+\frac{2v_a}{l}$

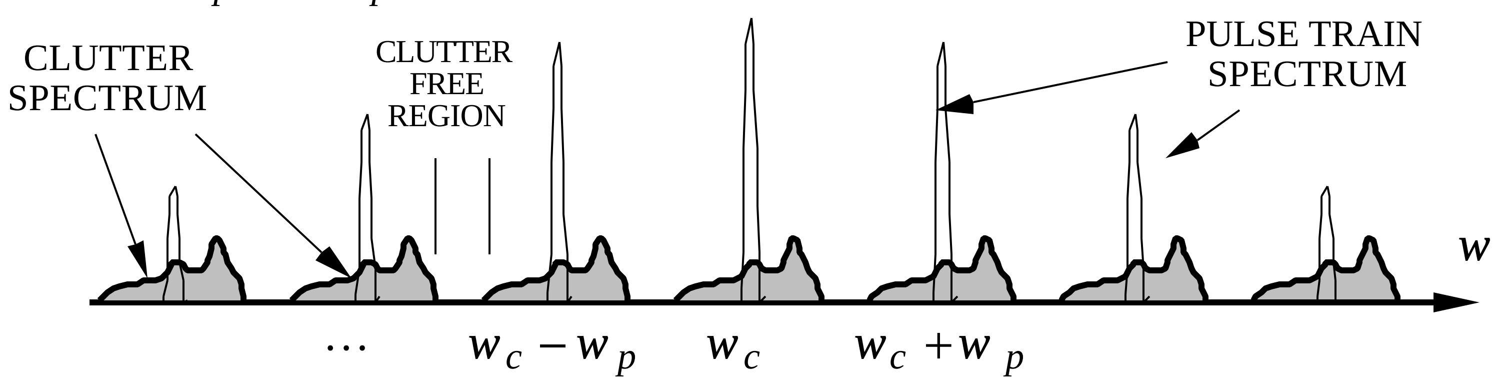
Clutter Spectrum (4)

Altitude clutter characteristics:

1. centered at $f_d = 0$ (unless aircraft is maneuvering)
2. slant ranges can be short at sidelobe angles but F_g can be large

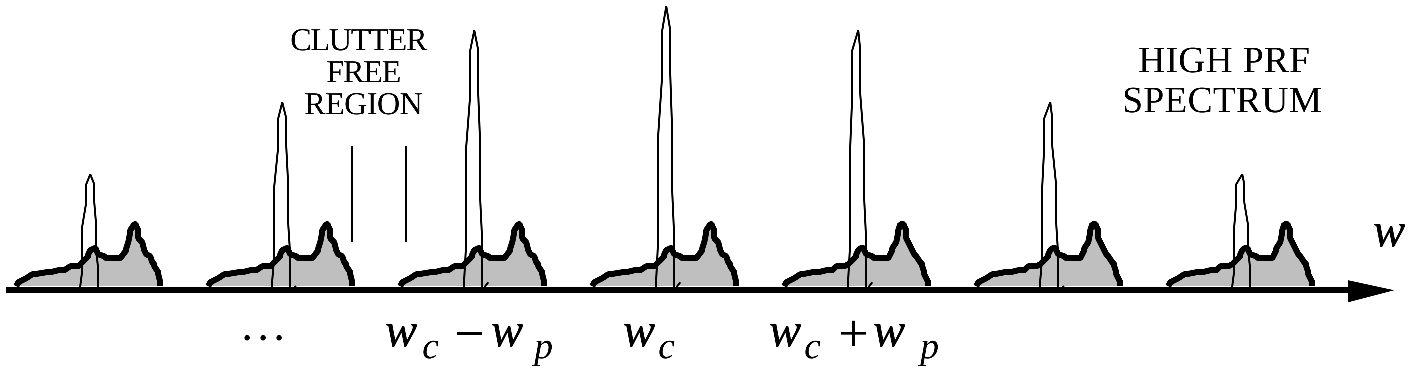


When the transmitted waveform is pulsed, the clutter spectrum repeats at multiples of the PRF $(w_p = 2p\ f_p)$

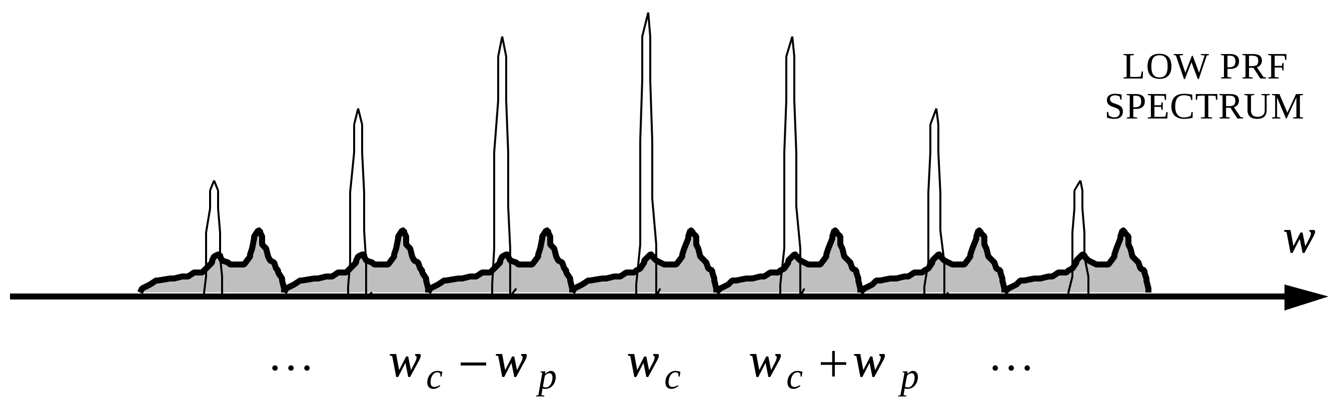


Clutter Spectrum (5)

High PRF -- clutter-free region



Low PRF -- clutter spectra overlap; no clutter-free region



Clutter Spectrum (6)

Example: low PRF radar with:

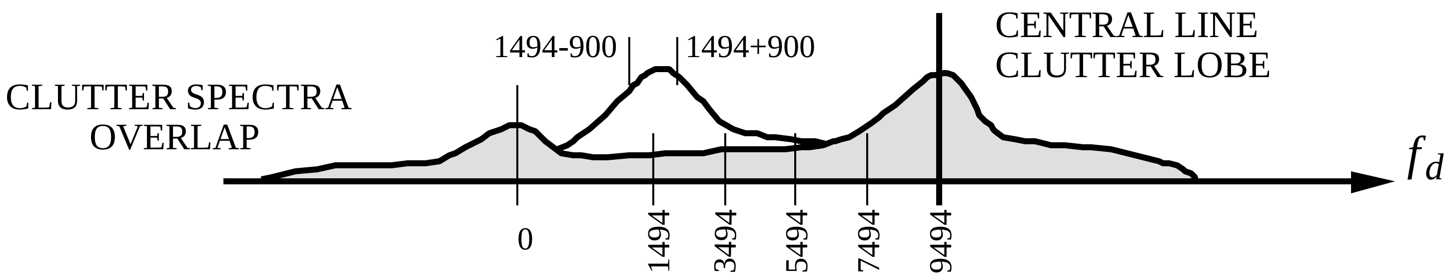
$$f = 9.5\,\text{GHz}, \quad v_a = 300\text{ m/s}, \quad q_B = 2.5^\circ, \quad g = 60^\circ, \quad \text{PRF} = 2\text{ kHz}$$

Unambiguous range and velocity:

$$R_u = \frac{c}{2 f_p} = \frac{3\times 10^8}{2(2000)} = 75\text{ km}; \qquad \Delta v_u = \frac{\pm f_{d\max}\, l}{2} = \frac{f_p\, l}{2}$$

$$\text{BW}_{\text{MLC}} = \frac{2v_a}{l}\sin(60^\circ)(2.5)\underbrace{(0.0436)}_{q_B} = 1800\text{ Hz}; \qquad = \frac{2000(0.0316)}{2} = 32\text{ m/s}$$

$$\text{BW}_{\text{SLC}} = 2\left(\frac{2v_a}{l}\right) = \frac{4(300)}{0.0316} \approx 38\text{ kHz}; \qquad f_{\text{MLC}} = \frac{2v_a}{l}\cos(60^\circ) \approx 9494\text{ Hz}$$



Clutter Spectrum (7)

high PRF radar with:

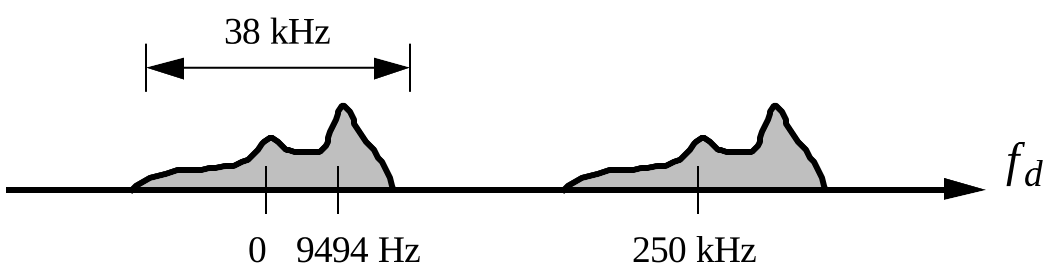
$$f = 9.5\,\text{GHz}, \quad v_a = 300\text{ m/s}, \quad q_B = 2.5^\circ, \quad g = 60^\circ, \quad \text{PRF} = 250\text{ kHz}$$

Unambiguous range and velocity:

$$R_u = \frac{c}{2 f_p} = \frac{3 \times 10^8}{2(250000)} = 600\text{ m}$$

$$\text{BW}_{\text{MLC}} = 1800\text{ Hz}; \qquad \Delta v_u = 3950\text{ m/s}$$

$$\text{BW}_{\text{SLC}} \approx 38\text{ kHz}; \qquad f_{\text{MLC}} = 9494\text{ Hz}$$



Sea States*

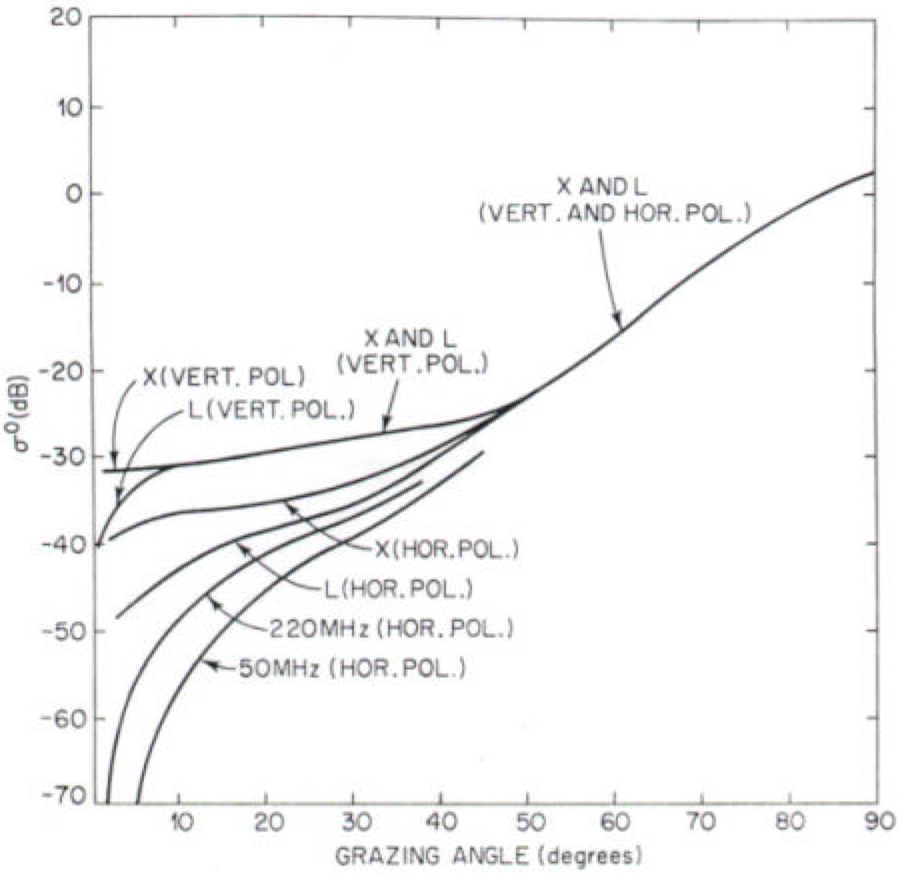
SEA STATE	WAVE HEIGHT (ft)	WAVE PERIOD (sec)	WAVE LENGTH (ft)	WAVE VELOCITY (kt)	PARTICLE VELOCITY (ft/sec)	WIND VELOCITY (kt)	REQUIRED FETCH (mi)
0 (flat)	(not a recognized sea state but often used to denote a flat sea)						
1 (smooth)	0-1	0-2	0-20	0-6	0-1.5	0-7	0-25
2 (slight)	1-3	2-3.5	20-65	6-11	1.5-2.8	7-12	25-75
3 (moderate)	3-5	3.5-4.5	65-110	11-14	2.8-3.5	12-16	75-120
4 (rough)	5-8	4.5-6	110-180	14-17	3.5-4.2	16-19	120-190
5 (very rough)	8-12	6-7	180-250	17-21	4.2-5.2	19-23	190-250
6 (high)	12-20	7-9	250-400	21-26	5.2-6.7	23-30	250-370
7 (very high)	20-40	9-12	400-750	26-35	6.7-10.5	30-45	370-600
8 (precipitous)	> 40	> 12	> 750	> 35	> 10.5	> 45	> 600

Note:
1. Assumes deep water.
2. Wave velocity determines clutter doppler; particle velocity determines how fast a particle moves.
3. Data only applies to waves; swells are generated at long distances by other wind systems.
4. Period, wavelength and wave velocity apply to swells and waves.
5. Fetch is the distance that the wind is blowing; duration is the length of time.

*After Edde, *Radar*, Prentice-Hall (also see Skolnik, Table 7.2)

Sea Clutter

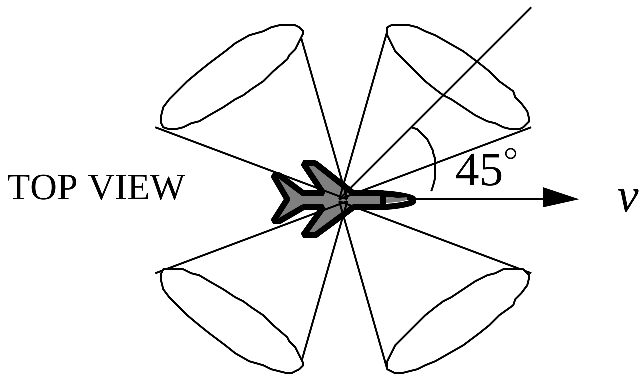
Composite of s^{o} data for average conditions with wind speed ranging from 10 to 20 knots (Fig. 7.3, Skolnik)

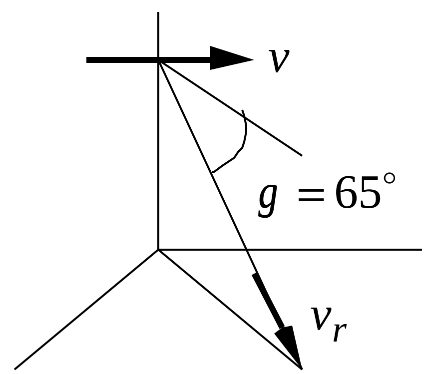


Example: AN/APS-200

The AN/APS-200 is a doppler navigation radar (see Skolnik p. 94 and *Microwaves*, October 1974). Radar parameters:

velocity range = 50 to 1000 knots; altitude = 0 to 70000 feet
vertical plane (elevation) beamwidth = 2.5 degrees
horizontal plane (azimuth) beamwidth = 5 degrees
$G = 30$ dB, $P_t = 1$ W (CW), $f = 13.3$ GHz ($l = 0.0225$ m)





The relative velocity is $v_r = 1000\cos(65^\circ)\cos(45^\circ) = \pm 299$ knots, or

$$f_d = \frac{2v_r}{l} = \frac{1.03v_r}{l} = \frac{1.03(299)}{0.0225} = 13.6\ \text{kHz}$$

Example: AN/APS-200

(a) Required bandwidth: $\mathrm{BW} = 2(13.6) = 27.3\ \mathrm{kHz}$

(b) P_r for $h = 40000$ feet over the ocean

For the ocean, 10 to 20 kt winds (average conditions): $s^o = -15\ \mathrm{dB}$

$$C = P_r = \frac{P_t G_t A_{er} A_c s^o}{(4p)^2 R^4}$$

For the beamwidth limited case (CW):

$$A_c = \frac{p R^2 q_{B_{\mathrm{el}}} q_{B_{\mathrm{az}}}}{4 \sin g}$$

For the identical transmit and receive antennas: $A_{er} = \dfrac{l^2 G}{4p} = 0.0407\ \mathrm{m}^2$. Therefore

$$C = \frac{P_t G_t A_{er} s^o}{(4p)^2 R^4} \frac{p R^2 q_{B_{\mathrm{el}}} q_{B_{\mathrm{az}}}}{4 \sin g} = \frac{(1)(10^3)(0.0407)(0.0316)(5)(2.5)\overbrace{(0.0174)^2}^{\text{CONVERT TO RADIANS}}}{64p(13.43\times 10^3)^2 \sin 65^\circ}$$

$$C = 1.49\times 10^{-13}\ \mathrm{W} = -128\ \mathrm{dBW} = -98\ \mathrm{dBm}$$

Example: AN/APS-200

(c) For the following parameters:

$$h = 40000 \text{ feet}$$
$$T_a = 300 \text{ K}$$
$$L = 5 \text{ dB}$$
$$F = 10 \text{ dB}$$
$$B_n \text{ from part (a)}$$

we can compute the clutter-to-noise ratio as follows:

$$\left(\frac{S}{N}\right)_{\text{out}} = \frac{S_{\min}}{N_{\text{out}}} = \frac{S_{\min}}{kT_o\left(\frac{T_a + T_e}{T_o}\right)B_n L}$$

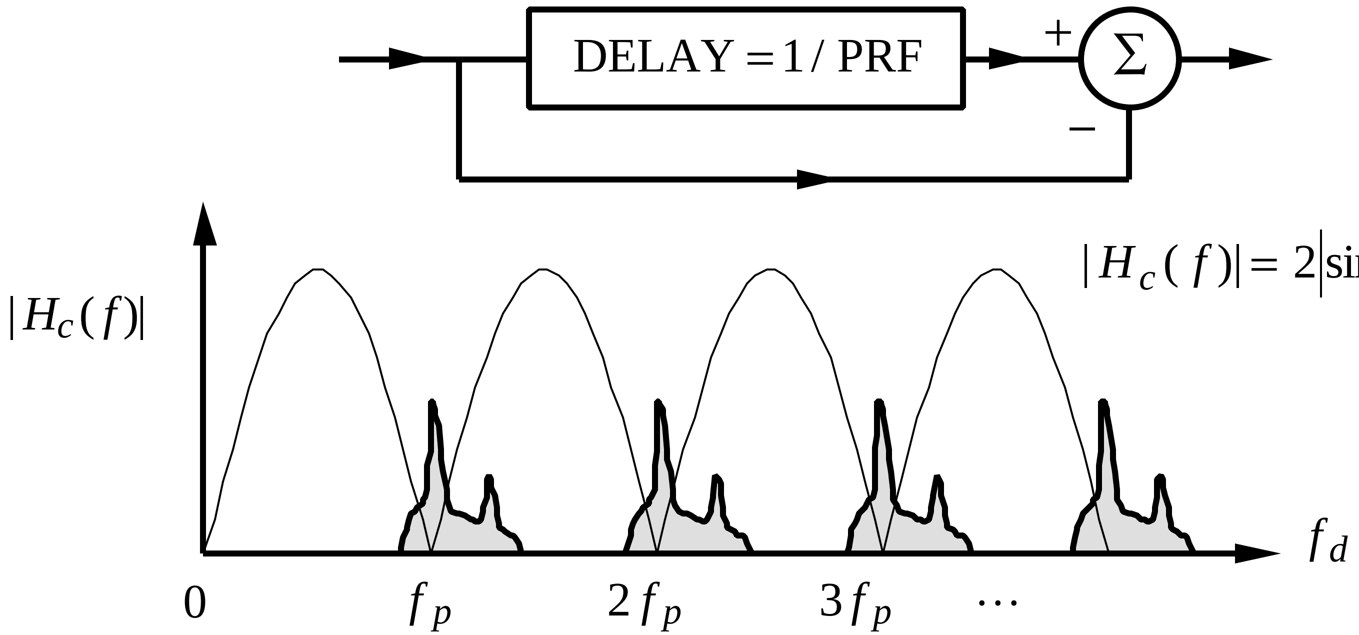
$$= \frac{1.49\times 10^{-13}}{kT_o\left(\frac{300+(10-1)(290)}{290}\right)B_n L}$$

$$= \frac{1.49\times 10^{-13}}{1.38\times 10^{-23}(2910)(27.3\times 10^{3})(3.16)}$$

$$= 47.95 \approx 17 \text{ dB}$$

Delay Line Canceler (1)

A delay line canceler is used to eliminate clutter. (It is also known as a transversal filter, taped delay line filter, non-recursive filter, moving average filter, and finite impulse response filter.)



It is effective if the clutter spectrum is narrow. Note that target returns with doppler frequencies in the notches are also rejected. These are blind speeds, which occur at

$$v_{bn} = n\,l\,f_p/2, \quad n = 0, \pm 1, \pm 2, \ldots$$

Delay Line Canceler (2)

Figures of merit for canceler performance:

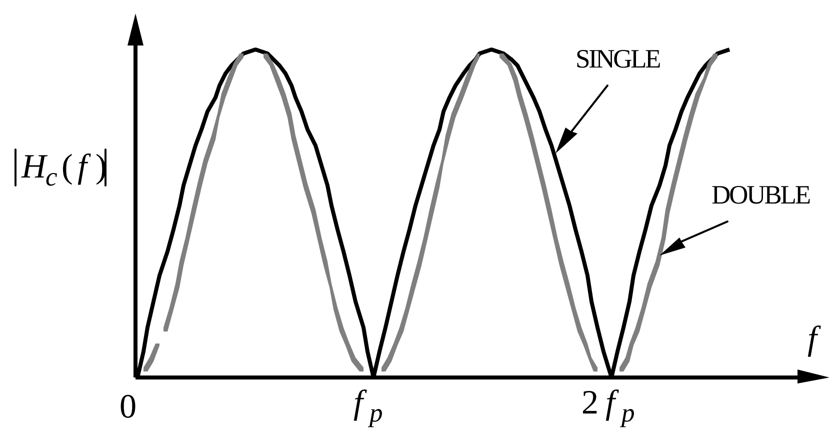
1. clutter attenuation -- where $S_c(w)$ is the clutter power spectral density

$$\text{CA} = \frac{\int_{-\infty}^{\infty} S_c(w)\,dw}{\int_{-\infty}^{\infty} S_c(w)\left|H_c(w)\right|^2 dw}$$

2. clutter improvement factor -- defined in terms of SCR, the signal-to-clutter ratio

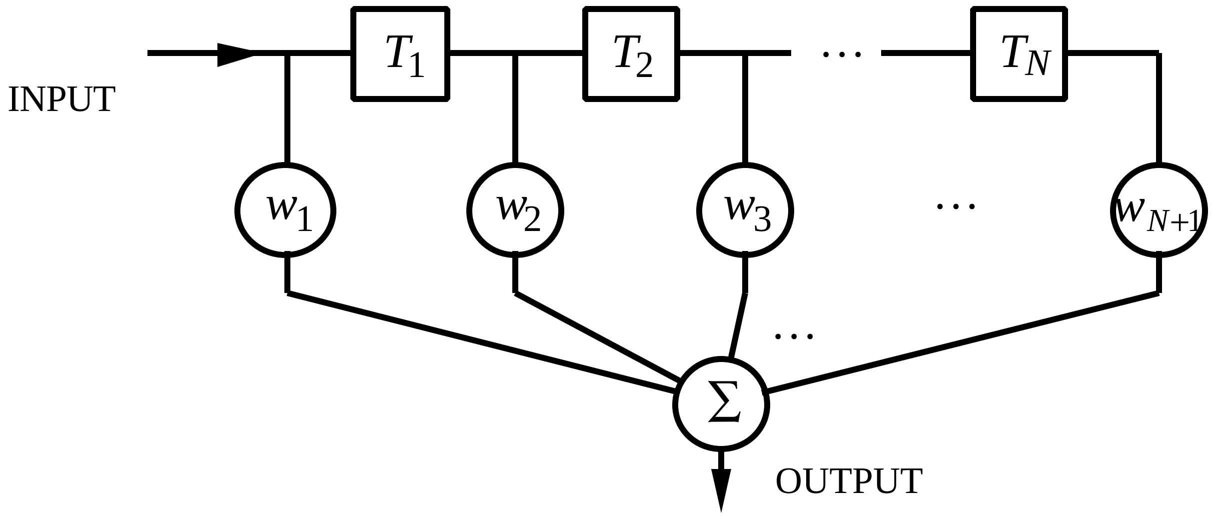
$$I_c = \frac{\text{SCR}_{\text{out}}}{\text{SCR}_{\text{in}}} = \frac{S_{\text{out}}}{S_{\text{in}}} \times \text{CA}$$

Double cancelers give a wider clutter notch



Delay Line Canceler (3)

Multiple pulse cancelers provide the ability to control the frequency characteristic



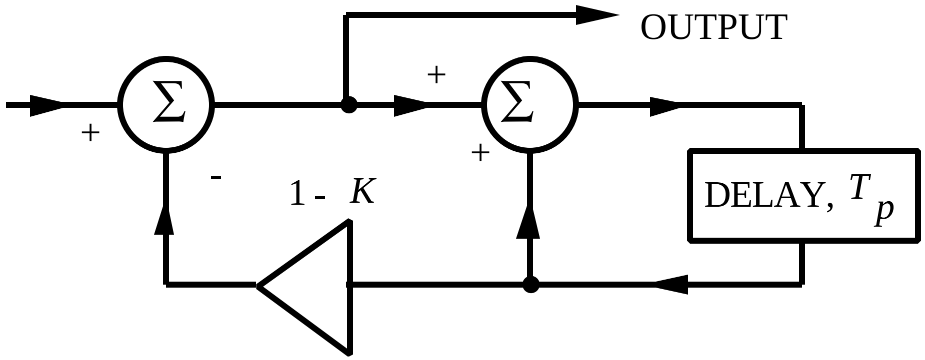
Weighting coefficients: binomial, Chebyshev, or optimum, which maximizes the clutter improvement factor, I_c. A N line canceler requires $N+1$ pulses, increasing the required time on target.

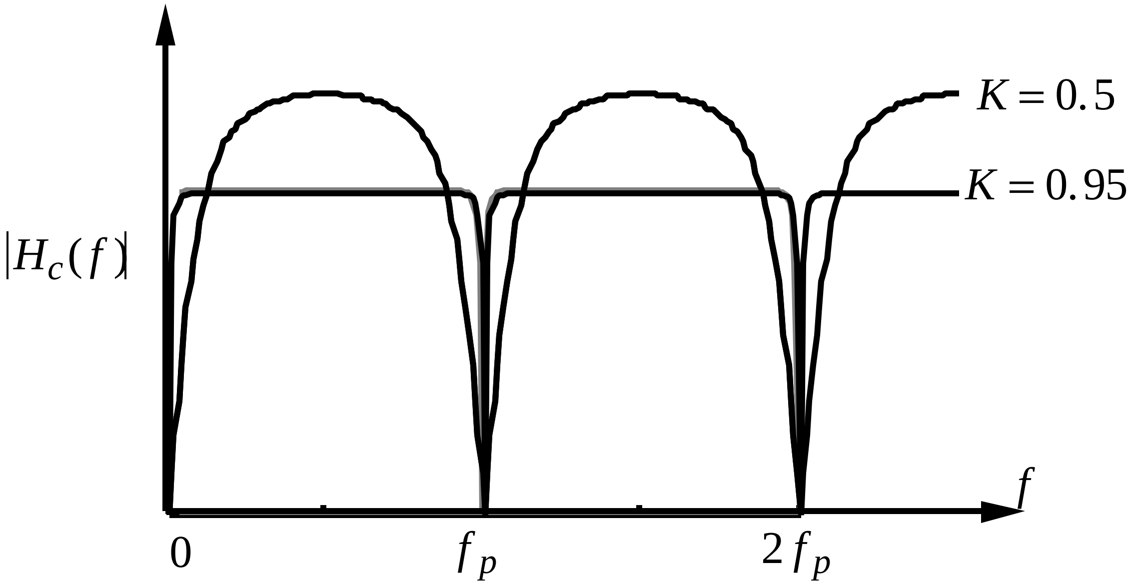
Delay line cancelers can be made recursive by adding a feedback loop. Frequency characteristics are of the form

$$|H_c(f)| = \frac{2\left|\sin\left(wT_p/2\right)\right|}{\sqrt{1+K^2-2K\cos(wT_p)}}$$

Delay Line Canceler (4)

Recursive single canceler and frequency characteristics

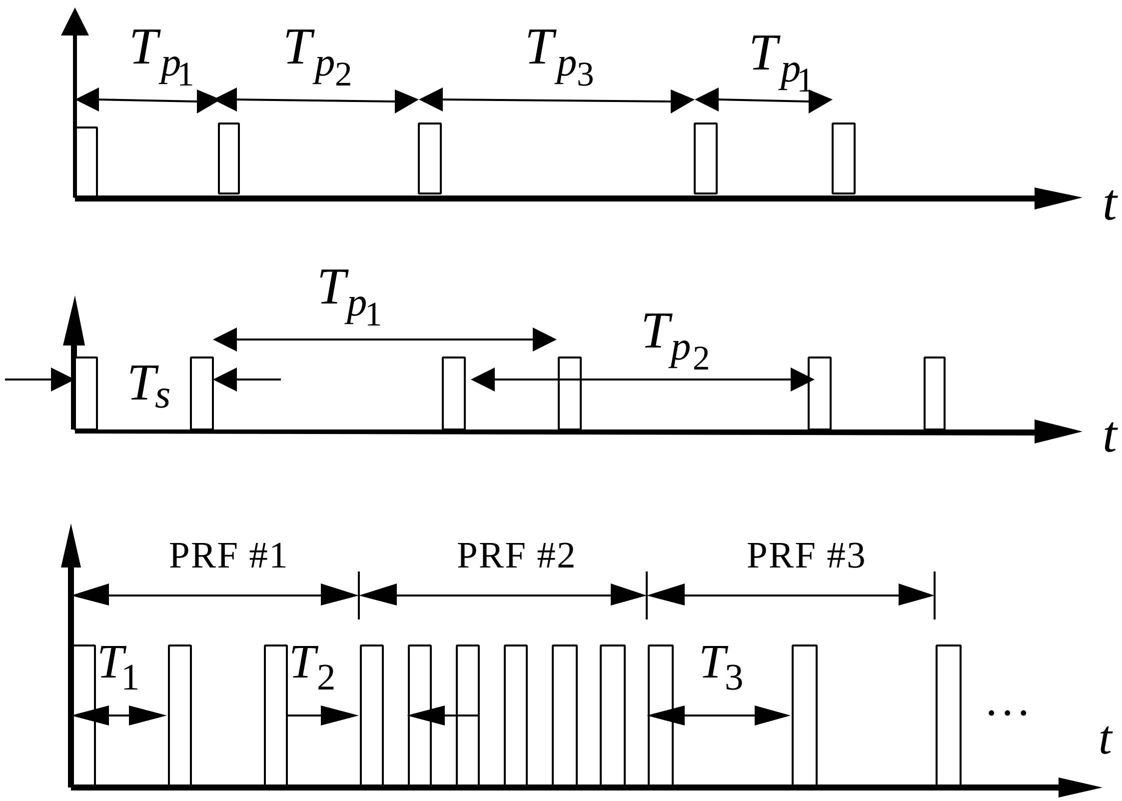




All delay lines suffer from blind speeds.

Staggered and Multiple PRFs (1)

The number of blind speeds can be reduced by employing multiple PRFs. They can be used within a dwell (look) or changed from dwell to dwell.



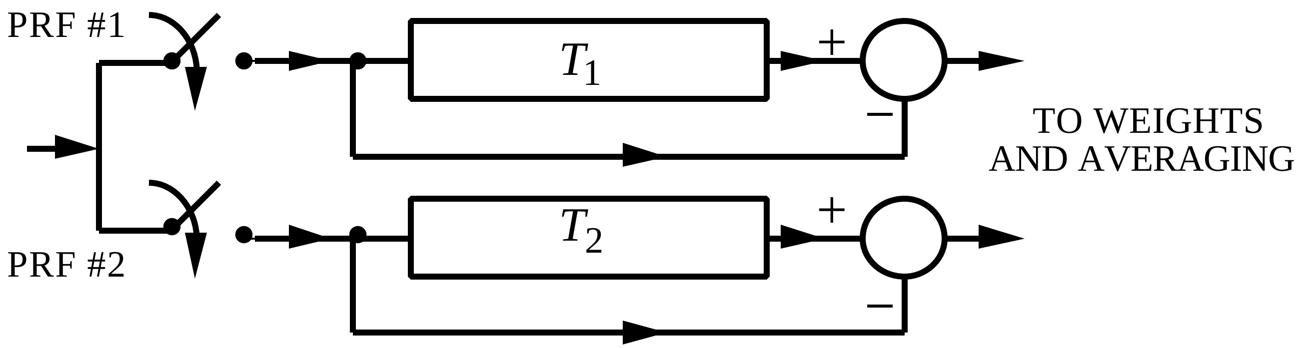
Staggered PRFs

Interlaced PRFs

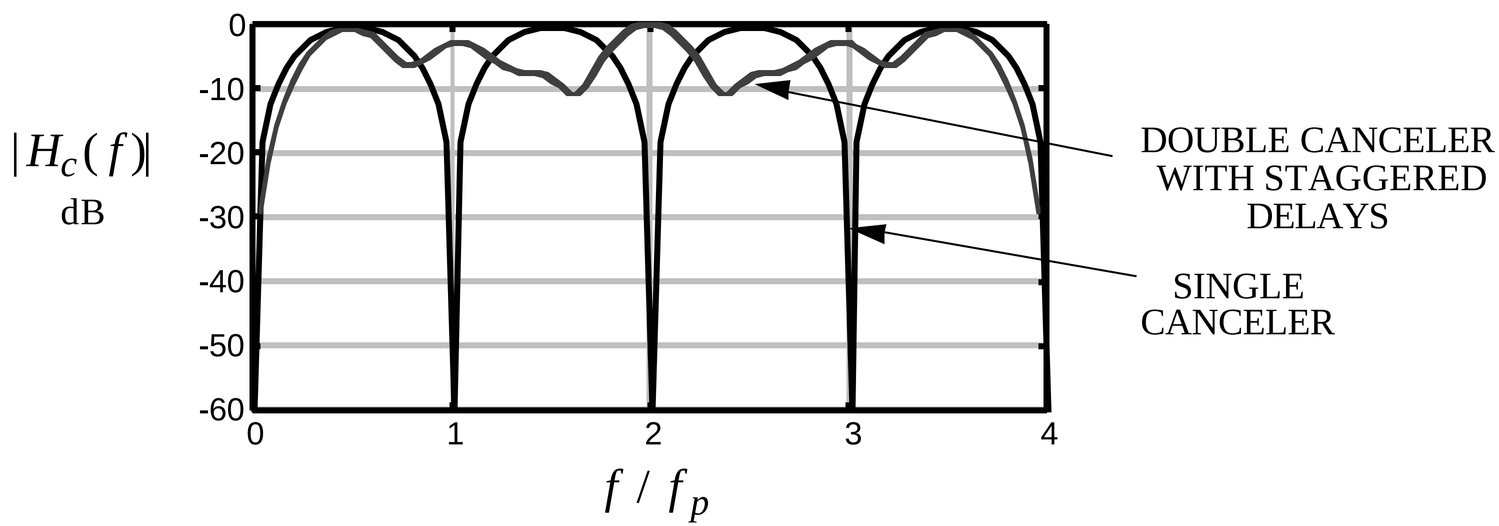
Multiple PRFs within a dwell

Staggered and Multiple PRFs (2)

Implementation of multiple PRFs:



Frequency characteristics



Staggered and Multiple PRFs (3)

Example: A MTI radar operates at 9 GHz and uses PRFs of 1 kHz and 1.25 kHz. What is the first blind speed?

Blind speeds for PRF #1:

$$v_n = \frac{n l}{2} f_{p1}$$

and for PRF #2:

$$v_m = \frac{m l}{2} f_{p2}$$

where m and n are integers. Both PRFs must have the same frequency at which the frequency characteristic is zero. Thus we require

$$\frac{n l}{2} f_{p1} = \frac{m l}{2} f_{p2} \quad \Rightarrow \quad n f_{p1} = m f_{p2} \quad \Rightarrow \quad \frac{m}{n} = \frac{f_{p1}}{f_{p2}} = \frac{4}{5}$$

The first blind speed ($n = 5$) is

$$v_1 = \frac{5l}{2} f_{p1} = \frac{5(0.033)}{2}(1000) = 83.33\text{m/s}$$

Synchronous Detection (I and Q Channels)

Synchronous detection uses I and Q channels. The complex signal representation of *s*

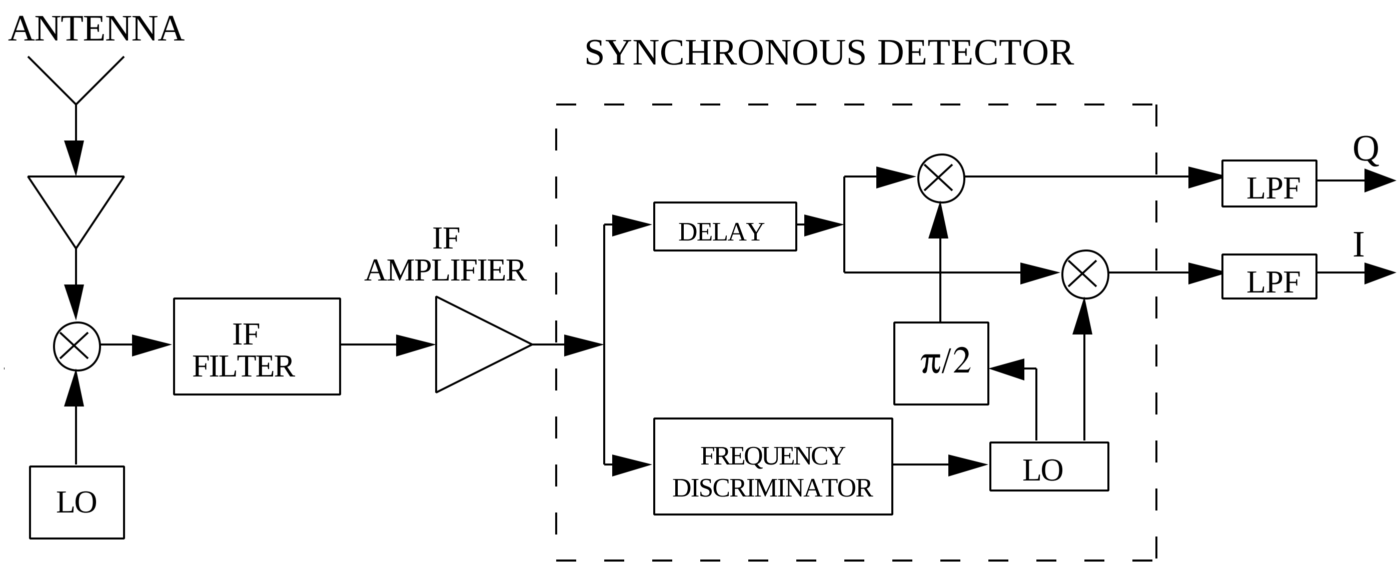
$$s = s_I - j s_Q \text{ and } |s| = \sqrt{s_I^2 + s_Q^2}$$

where

$$\mathrm{Re}\{s\} = |s| \cos\Phi_s \equiv s_I$$

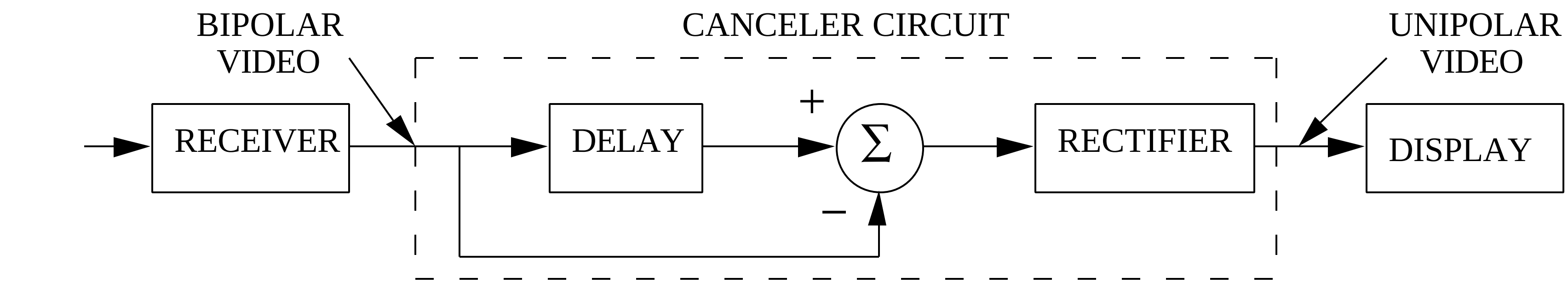
$$\mathrm{Im}\{s\} = |s| \sin\Phi_s = |s| \cos(\Phi_s - p/2) \equiv s_Q$$

Hardware implementation:

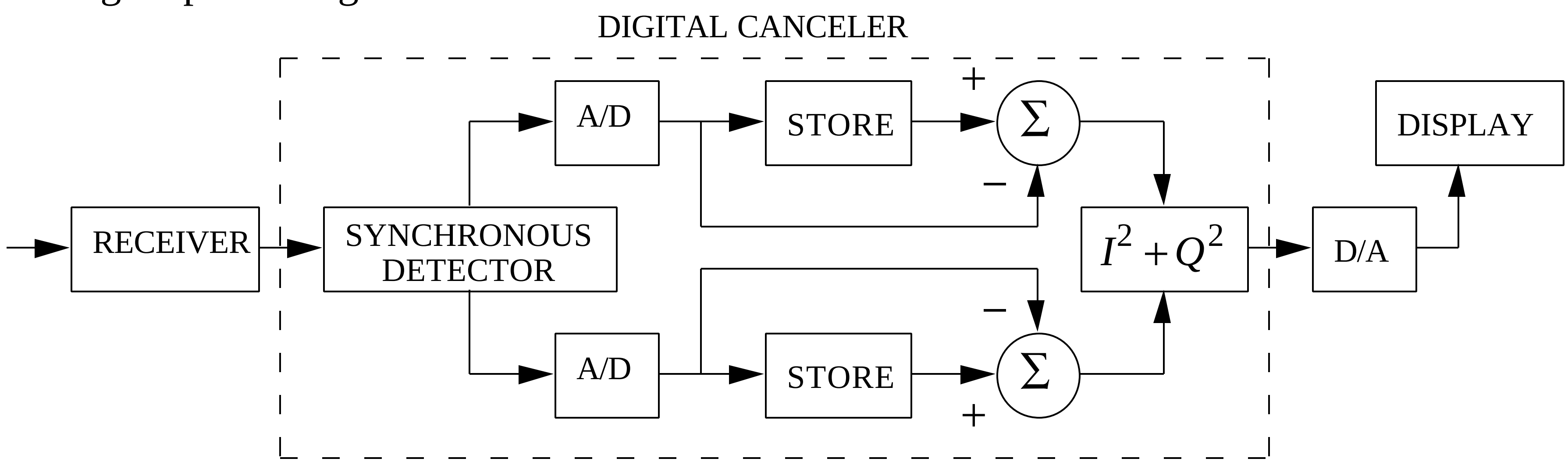


Analog vs. Digital Processing for MTI

Analog processing:

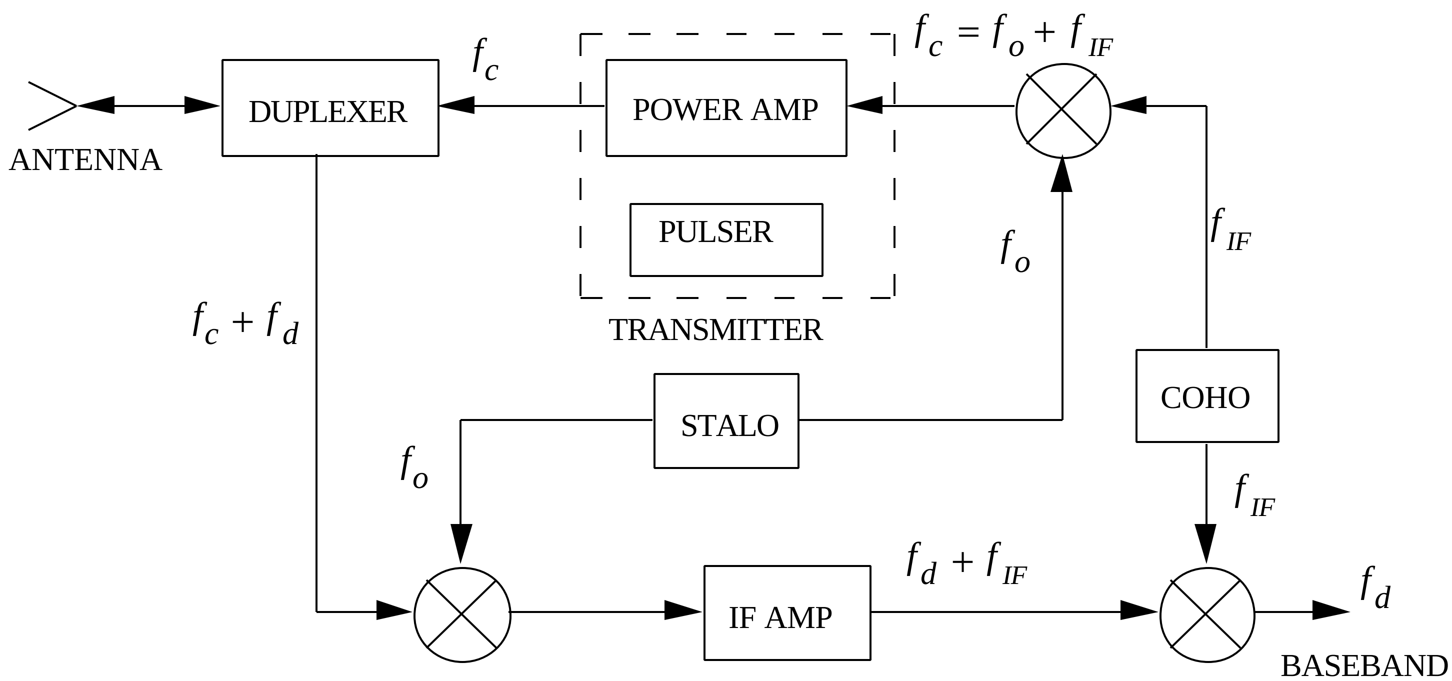


Digital processing:



Single Channel Receiver Block Diagram

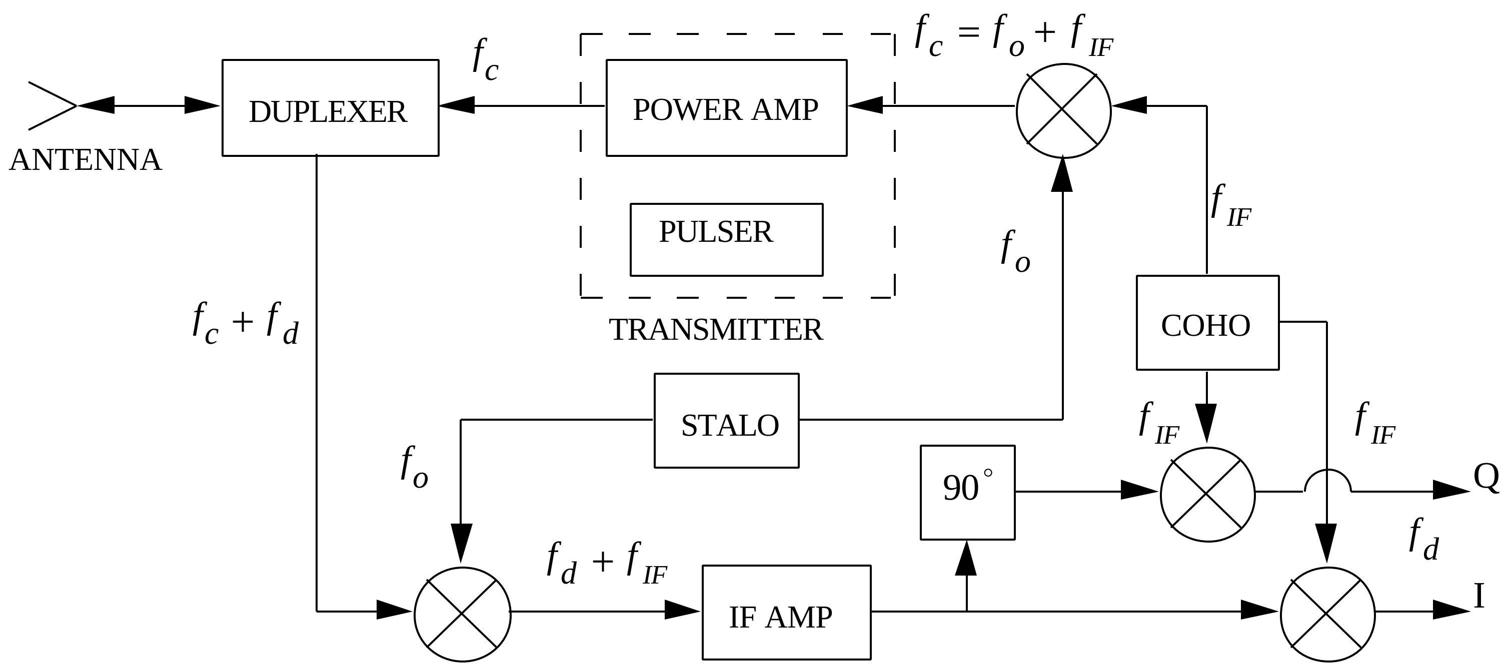
Block diagram of a single channel receiver (see Fig. 3.7 in Skolnik)



STALO = stable local oscillator
COHO = coherent oscillator

Synchronous Receiver Block Diagram

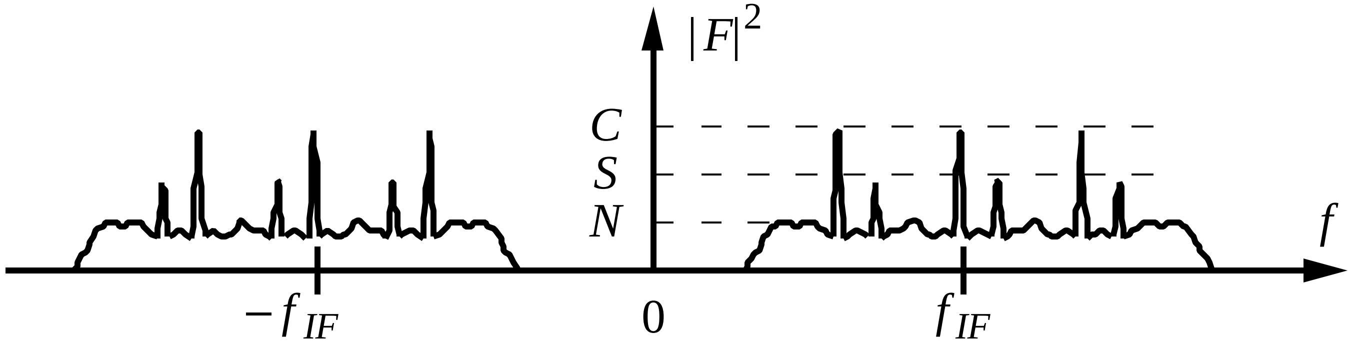
Block diagram of a synchronous receiver



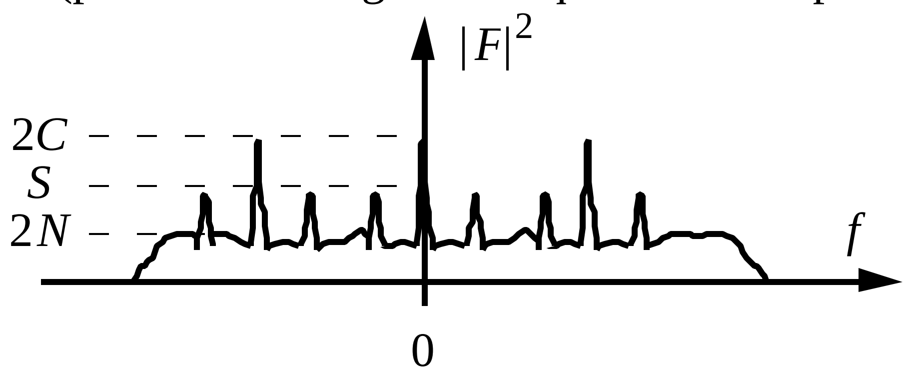
STALO = stable local oscillator
COHO = coherent oscillator
I = in phase component, $p(t)\cos(w_d t)$
Q = quadrature component, $p(t)\sin(w_d t)$

SNR Advantage of Synchronous Detection (1)

Single channel IF spectrum: (positive and negative frequencies are mirror images)



Single channel video: (positive and negative frequencies "wrap around")

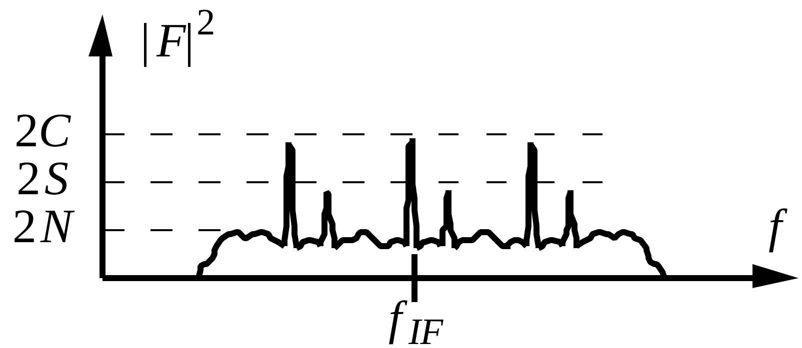


Signal-to-noise ratio:

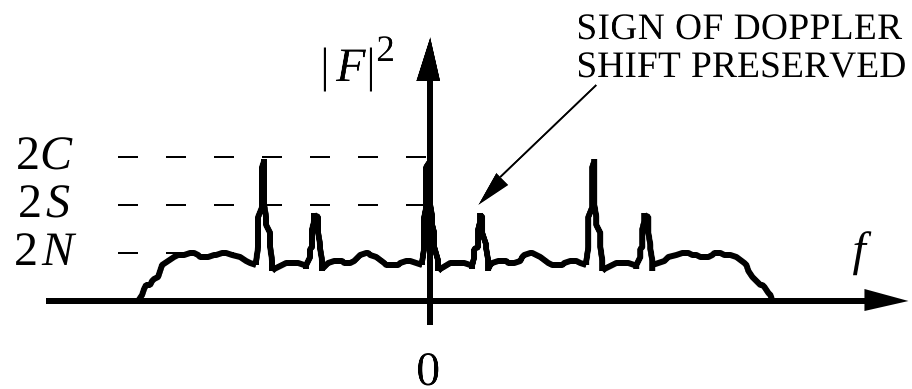
$$\mathrm{SNR} = \frac{S}{2N}$$

SNR Advantage of Synchronous Detection (2)

I/Q channel spectrum: (no negative frequencies due to even/odd symmetry of the real/imaginary parts)



I/Q channel baseband:



Signal-to-noise ratio:

$$\mathrm{SNR} = \frac{2S}{2N} = \frac{S}{N}$$

Processing of a Coherent Pulse Train (1)

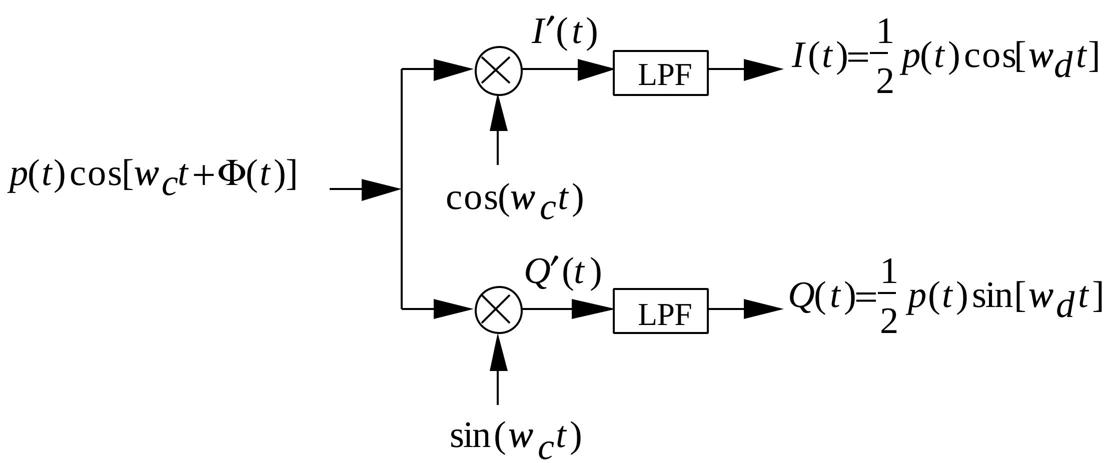
Assume synchronous detection at the carrier frequency. The received signal will be of the form

$$s(t) = p(t)\cos[w_c t + \Phi(t)]$$

where $p(t)$ is the pulse train envelope and $\Phi(t) = w_d t$ $(w_d = 2p\ f_d)$. Signals into the filters are

$$I'(t) = \frac{1}{2}\left\{p(t)\cos[2w_c t + \Phi(t)] + p(t)\cos[\Phi(t)]\right\}$$
$$Q'(t) = \frac{1}{2}\left\{p(t)\sin[2w_c t + \Phi(t)] + p(t)\sin[\Phi(t)]\right\}$$

Filtering removes the first terms in the brackets.



Sampling Theorem (1)

Consider a waveform $s(t) \leftrightarrow S(w)$ which is bandlimited ($S(w) = 0$ for $|w| \geq w_o$). The the function $s(t)$ can be uniquely determined from the values

$$s_n = s(np / w_o)$$

These are samples spaced at intervals of $p / w_o = 1/(2 f_o)$, which is twice the highest frequency contained in the bandlimited signal. The waveform is reconstructed from

$$s(t) = \sum_{n=-\infty}^{\infty} s_n \,\mathrm{sinc}(w_o t - np)$$

Real-world signals are modeled as bandlimited even though they rarely are. (They can be approximated arbitrarily closely by bandlimited functions.) Delta functions are often used as sampling functions. Multiplying a waveform by an infinite series of Dirac delta functions is an ideal sampling process

$$s(t_n) = \sum_{n=-\infty}^{\infty} s(t) d(t - nT_s)$$

where T_s is the sampling interval.

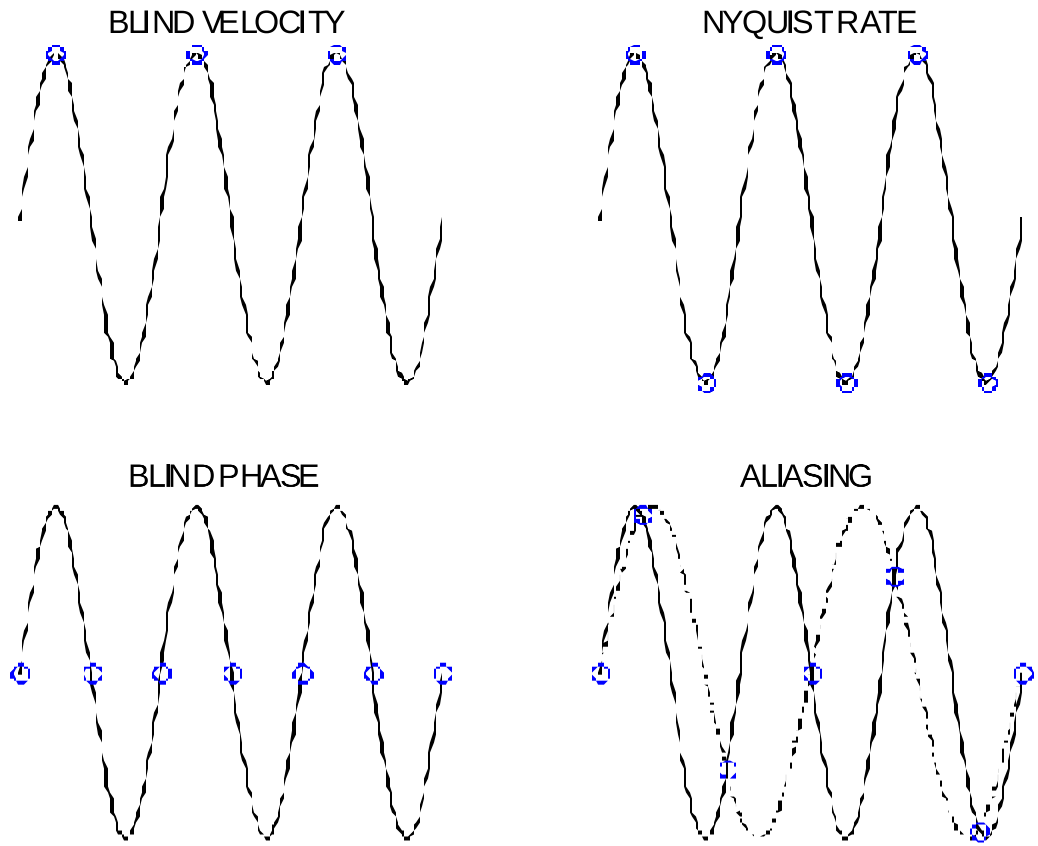
Sampling Theorem (2)

The Nyquist sampling rate (twice the highest frequency) is the minimum sampling rate that provides unique recovery. Other values result in undersampling or oversampling.

Some specific sampling cases
frequency f_o

$$f_s = 1/T_s \begin{cases} = 2f_o, & \text{Nyquist rate} \\ > 2f_o, & \text{oversampling} \\ < 2f_o, & \text{undersampling} \end{cases}$$

(o is a sample point)



Processing of a Coherent Pulse Train (2)

Simplifications:
1. neglect noise, clutter, etc.
2. assume that the target stays in the same range bin for all pulses in the train

The signal return from a target is reconstructed from returns with a constant delay after each pulse (i.e., same range bin)

Sample once per pulse: the sampling frequency is f_p, the sampling times $t_1, t_2, \ldots, t_N$

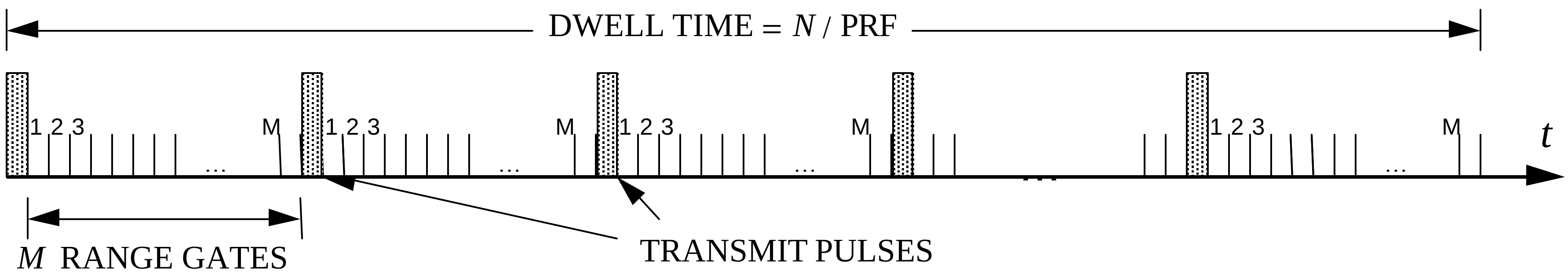
$$I(t_n) = \frac{1}{2}\left\{p(t_n)\cos[w_d t_n]\right\}$$
$$Q(t_n) = \frac{1}{2}\left\{p(t_n)\sin[w_d t_n]\right\}$$

This signal is equivalent to the complex form:

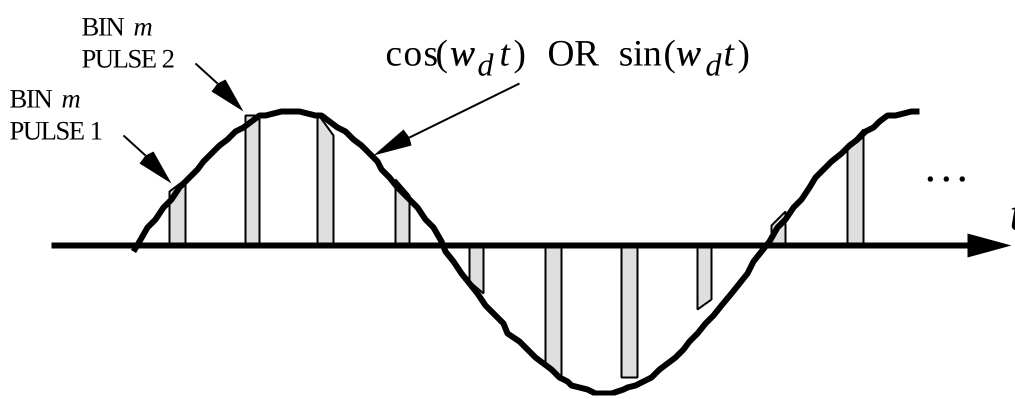
$$f(t_n) = 2\left\{I(t_n) + jQ(t_n)\right\} = p(t_n)\,e^{j\Phi(t_n)} = p(t_n)\,e^{jw_d t_n}$$

Processing of a Coherent Pulse Train (3)

Pulse train with range gates (bins):

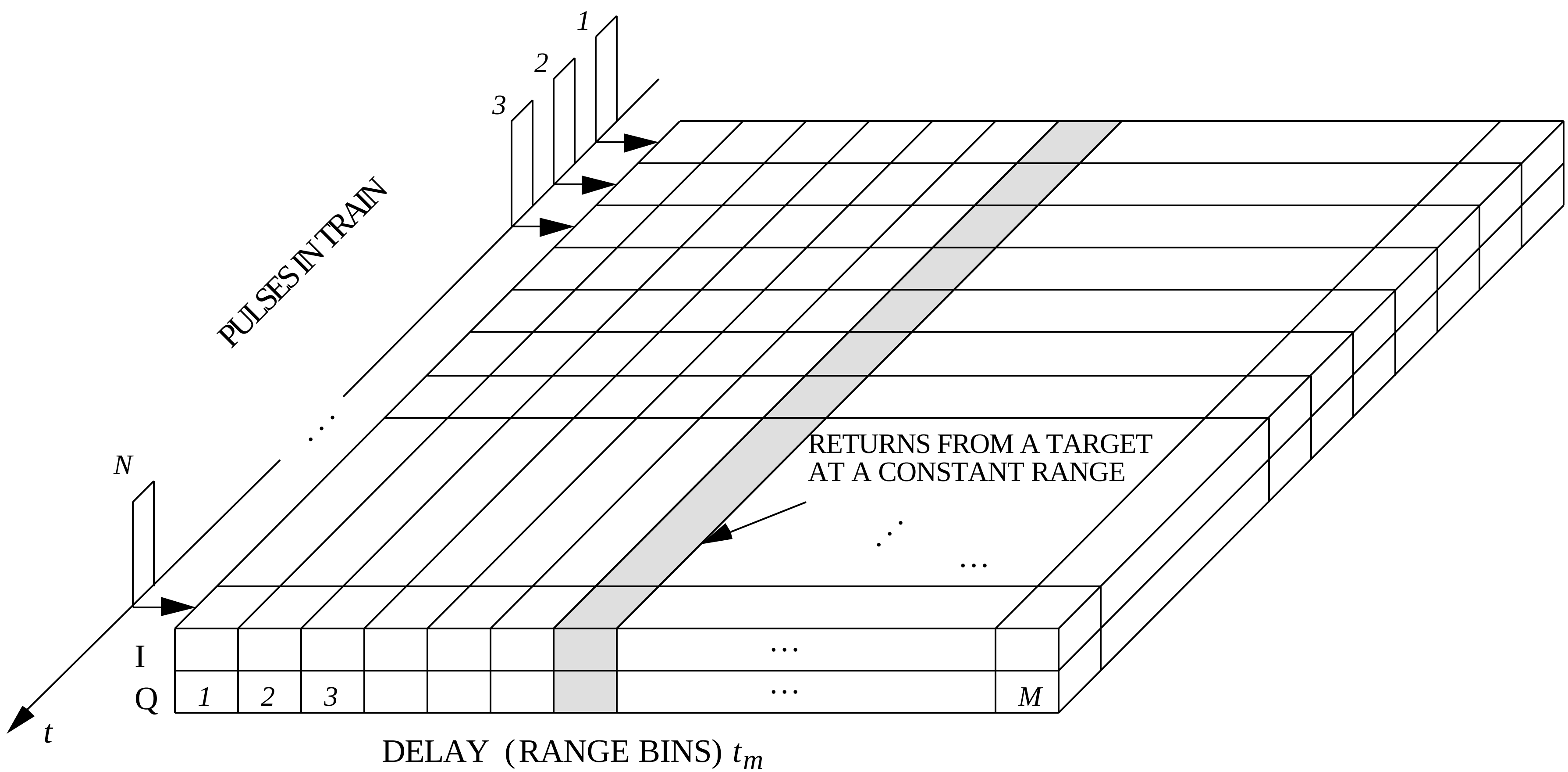


Returns for a target that remains in a single range bin (for example, bin m)



Processing of a Coherent Pulse Train (4)

Storage of data in a two-dimensional array

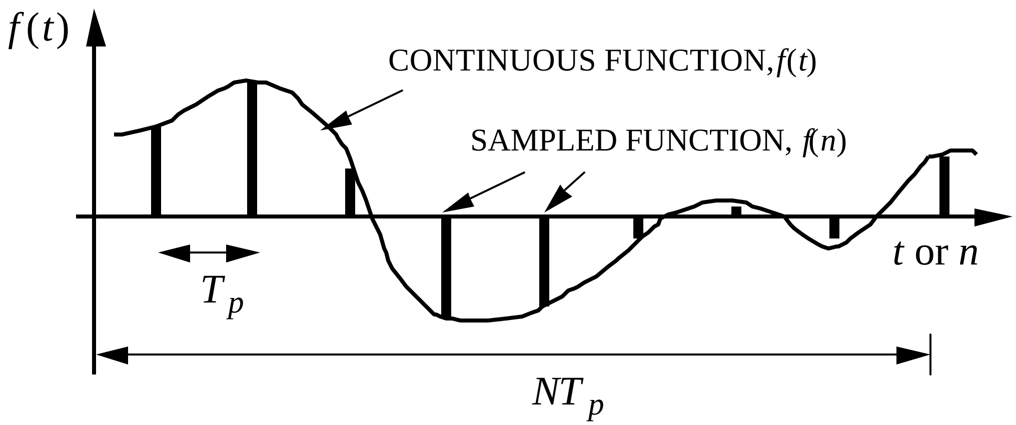


Discrete Fourier Transform (DFT)

The discrete Fourier transform is a sampled version of the conventional Fourier transform defined by

$$F(w_k) = \sum_{n=0}^{N-1} f(t_n) e^{-j\left(\frac{2p}{N}\right)kn} \qquad (k = 0,1,\ldots N-1)$$

where $w_k = \dfrac{2p}{NT_p}k$ and $t_n = nT_p$. $F(k)$ and $f(n)$ are sometimes used to denote sampled data.



Sampling rate: $f_s = 1/T_p \equiv f_p$

Frequency resolution: $\Delta f = 1/(NT_p)$ (signal duration is NT_p)

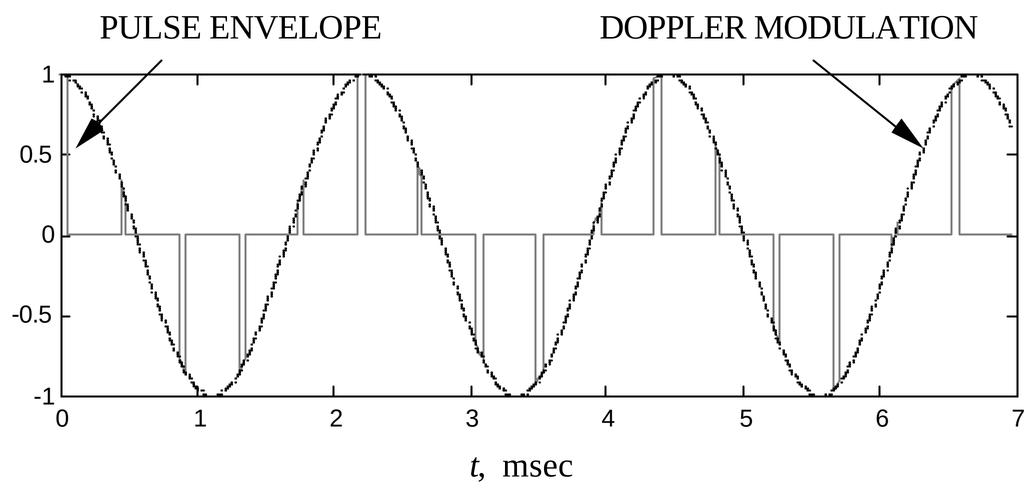
For unambiguous frequency measurement: $f_s \geq 2 f_{\max}$, or $f_{\max} \leq f_s/2$

Doppler Filtering Using the DFT (1)

Assume that there is a target return in a fixed range bin (e.g., #2):

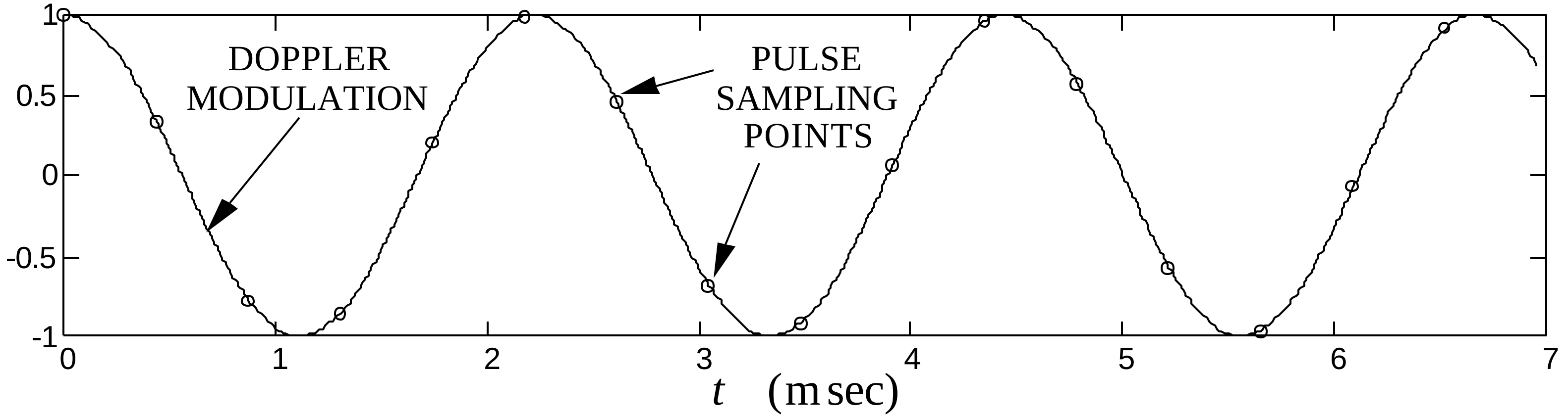
1. There are N pulses in a dwell and therefore the sinusoid is sampled N times
   - -- the sampling rate is the PRF
   - -- each range bin gets sampled N times
   - -- there are a total of $N \times M$ data points per dwell
2. The data from each set of range bins is Fourier transformed. Typically the FFT is used, which requires that $N = 2^n$, where n is an integer.
3. The FFT returns N frequencies

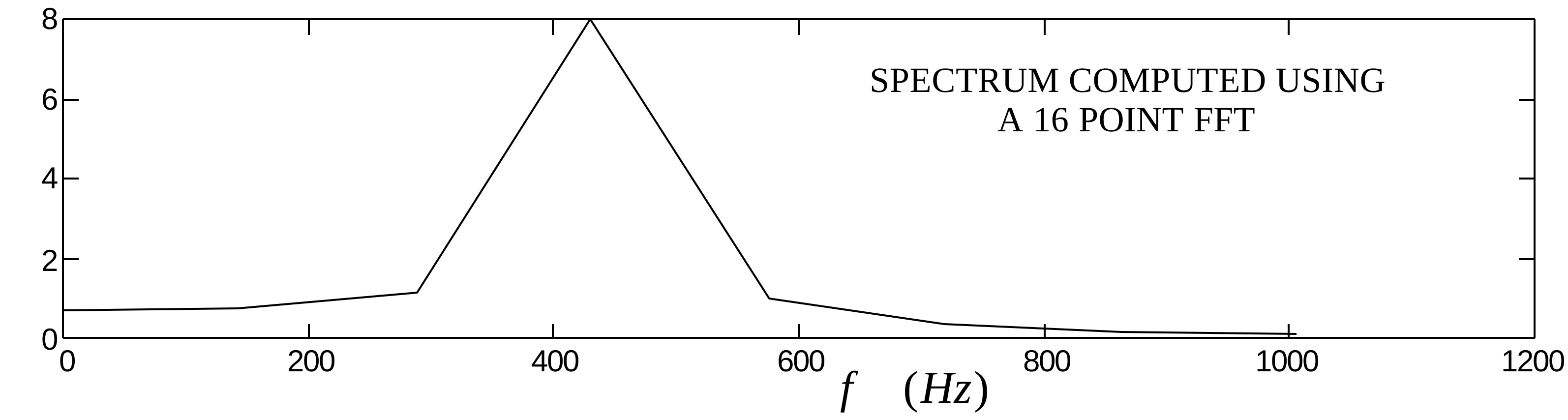
Example: pulse train ($N = 16$) echo return modulated by target doppler (ideal -- no dispersion or noise)



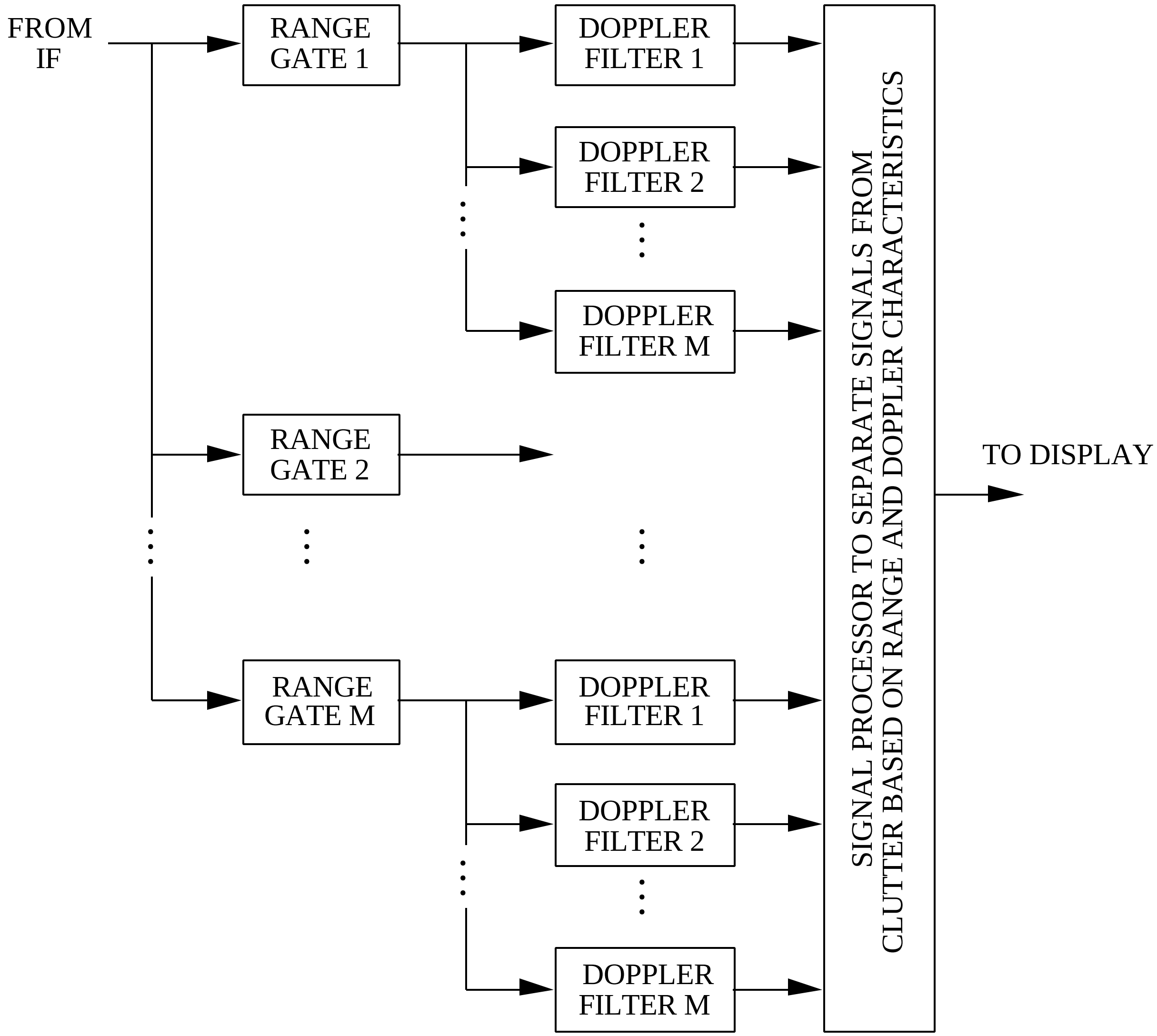
Doppler Filtering Using the DFT (2)

Calculation parameters: $f_d = 450$ Hz, $t = 150\ m\sec$, PRF=2300 Hz. From the parameters: $\Delta f = 1/(NT_p) = 143.8\,\text{Hz}$ $\quad f_s = 2300\,\text{Hz}$ $\quad NT_p = 6.957\,\text{ms}$



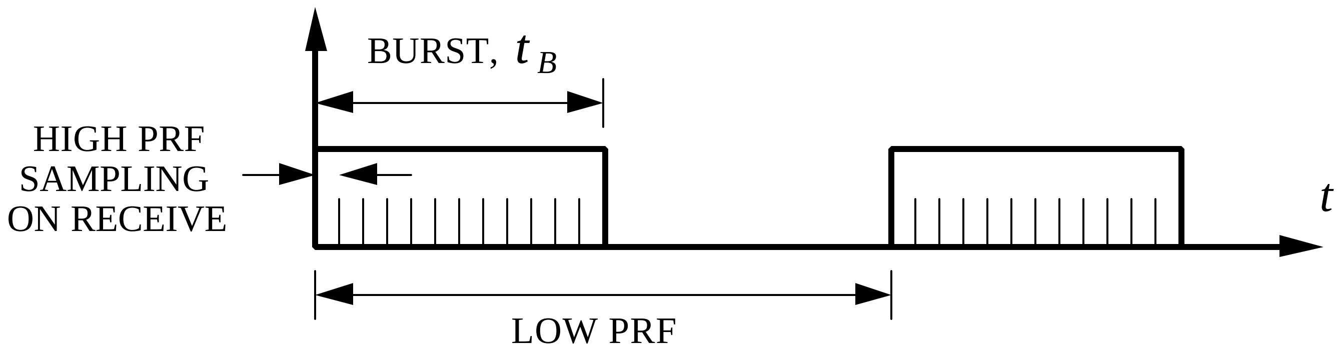


Pulse Doppler Receiver



Pulse Burst Mode

Pulse burst is a combination of low and high PRFs. A burst is a long pulse (of length t_B) that is transmitted at a burst repetition frequency (BRF) which has a low PRF value. On receive, the data is sampled at a HPF rate to avoid doppler ambiguities:



Typical steps:

1. Coarse range is measured by sorting the data into blocks of length $c t_B / 2$ ("burst delay ranging").
2. Samples are sorted by doppler by taking the FFT over the dwell of samples contained in each range block. Course doppler is obtained to within $1/t_B$.
3. Further processing of the data can improve the doppler measurement.

Tradeoffs: short burst $\Rightarrow$ lower competing clutter power
long burst $\Rightarrow$ higher SNR and smaller doppler bin size
computationally demanding (i.e., computer processing and memory)

MTI Improvement Factors

MTI improvement factor (clutter improvement factor):

$$I_c = \frac{\mathrm{SCR}_{\mathrm{out}}}{\mathrm{SCR}_{\mathrm{in}}} = \frac{\mathrm{S}_{\mathrm{out}}}{\mathrm{S}_{\mathrm{in}}} \times \mathrm{CA}$$

where SCR is the signal to clutter ratio.

Subclutter visibility: ratio by which the target return may be below the coincident clutter return and still be detected (with a specified P_d and P_{fa}).

Clutter attenuation:

$$\mathrm{CA} = \frac{\text{clutter power into canceler or filter}}{\text{clutter power remaining after cancelation}} = \frac{\int_{-\infty}^{\infty} S_c(w)\,dw}{\int_{-\infty}^{\infty} S_c(w)\,|H_c(w)|^2\,dw}$$

where $S_c(w)$ is the clutter spectrum and $H_c(w)$ the canceler/filter characteristic.

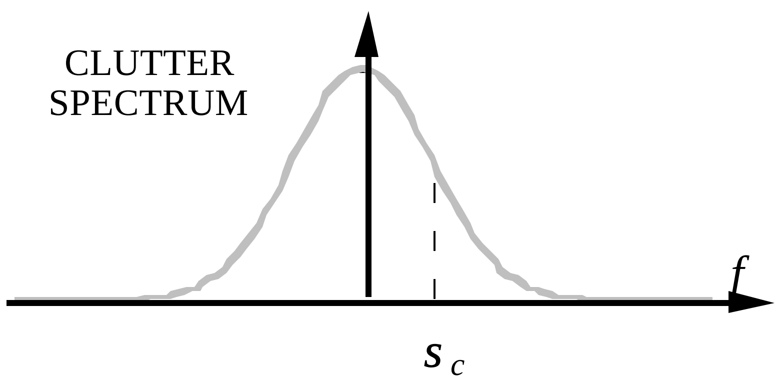
Cancellation ratio:

$$\mathrm{CR} = \left.\frac{\text{canceler voltage amplification}}{\text{gain of single unprocessed pulse}}\right|_{\substack{\text{ANTENNA AND}\\ \text{TARGET FIXED}}}$$

MTI Limitations (1)

Fluctuations in the clutter and instabilities in the radar system cause the clutter spectrum to spread. A simple model approximates the clutter spectrum as a gaussian with standard deviation s_c. If the contributing random processes are independent, then the total spectrum variance is

$$s_c^2 = s_F^2 + s_m^2 + s_w^2 + s_Q^2$$



1. Equipment instabilities: frequencies, pulsewidths, waveform timing, delay line response, etc. For transmitter frequency drift:

$$s_F = \frac{2.67}{B_n}\left(\frac{df}{dt}\right)$$

2. Quantization errors in digital processing: s_Q (rule of thumb: 6 dB per bit)

MTI Limitations (2)

3. Clutter fluctuations due to wind and motion: Clutter return is a random process. Let the standard deviation of the wind velocity be s_v. The corresponding standard deviation of the clutter spectrum due to wind motion is $s_w = 2s_v / l$

4. Antenna scanning modulation: the antenna periodically illuminates the target and therefore the return looks like a pulse train that gets switched off and on

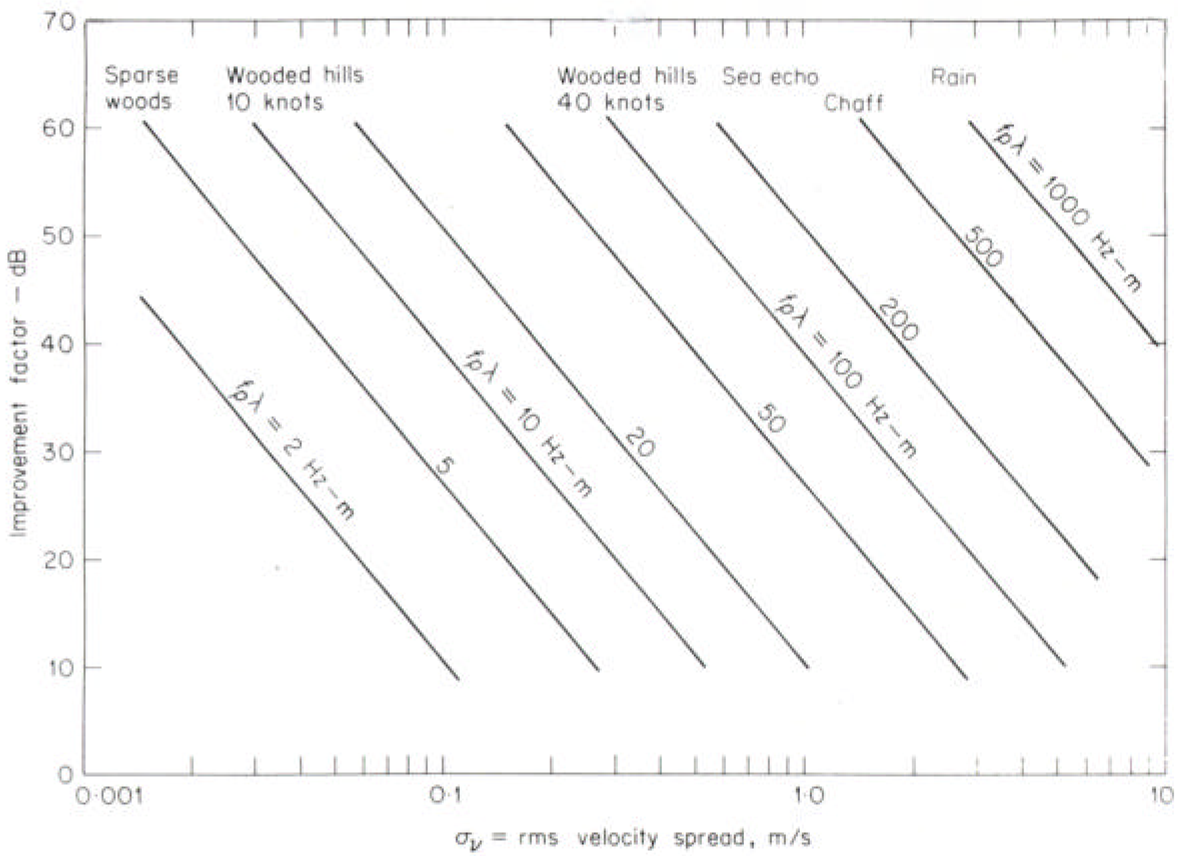
$$s_m = \frac{0.265 f_p}{n_B}$$

Note that:

1. A gaussian spectrum is maintained through the frequency conversion process and synchronous detection.
2. Envelope or square law detection doubles the variance

MTI Canceler Improvement Factors

Improvement factors for delay line cancelers (coherent, no feedback):



$$\text{Single}: \quad I_{c1} = 2\left(\frac{f_p}{2p s_c}\right)^2$$

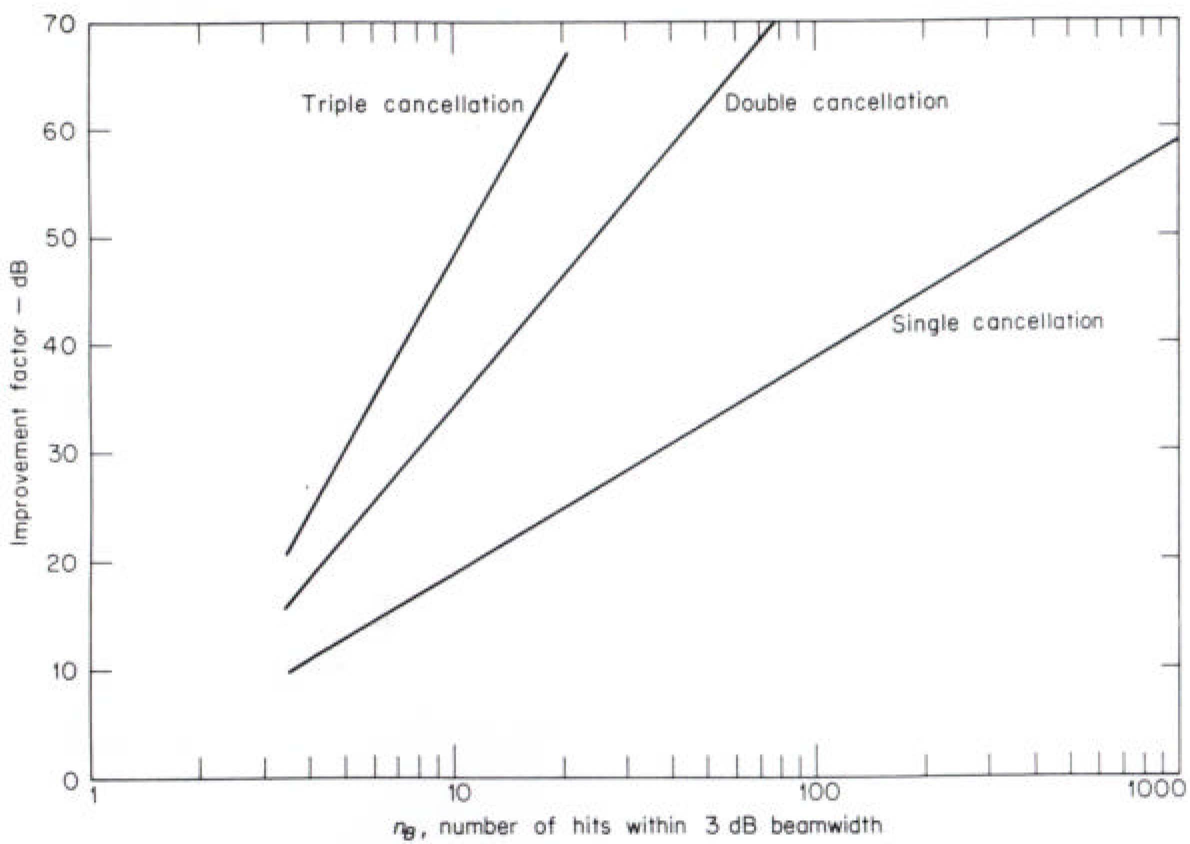
$$\text{Double}: \quad I_{c2} = 2\left(\frac{f_p}{2p s_c}\right)^4$$

Fig. 4.30 in Skolnik, 2nd edition

Chart for double canceler performance

MTI Canceler Improvement Factors

Improvement factors for delay line cancelers:



Single : $I_{s1} = \dfrac{n_B^2}{1.388}$

Double : $I_{s2} = \dfrac{n_B^4}{3.853}$

Fig. 3.32 in Skolnik

Chart for limitation due to antenna scanning modulation

Example

Example: A radar with $f = 16$ GHz ($l = 0.018$), $n_B = 10$ and $f_p = 530$ Hz using a double canceler and operating in wooded hills with 10 kt winds.

The improvement factor considering clutter motion: $s_v = 0.04 \Rightarrow s_c = s_w = \dfrac{2(0.04)}{0.018}$

$$I_{c2} = 2\left(\frac{f_p}{2p s_c}\right)^4 = 2\left(\frac{530}{2p(4.267)}\right)^4 = 3\times 10^6 = 54.9 \text{ dB}$$

The limitation in improvement factor due to antenna modulation if the antenna is scanning and there are n_B pulses hitting the clutter:

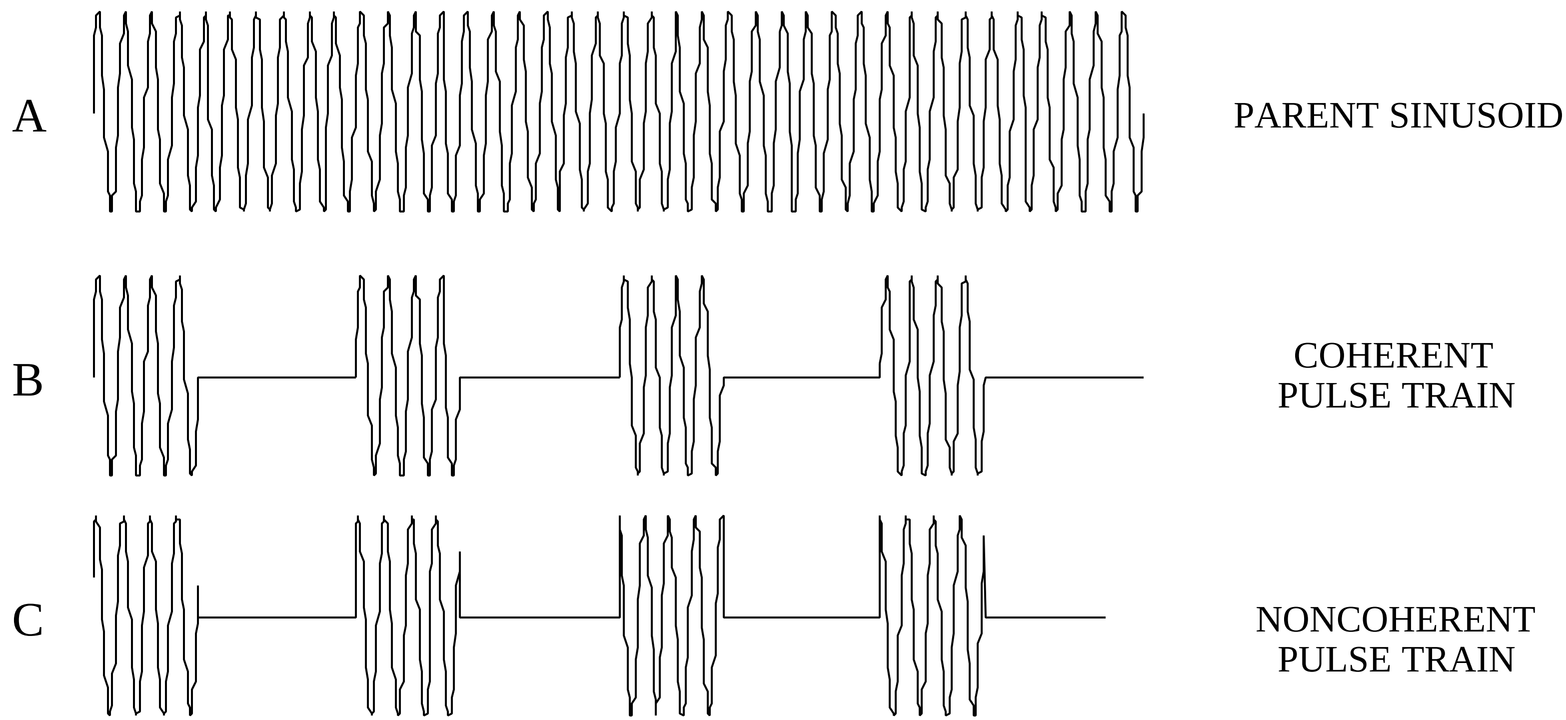
$$I_{s2} = \frac{n_B^4}{3.85} = 2597.4 = 34.1 \text{ dB}$$

Note that a similar result would be obtained by including the antenna modulation in the calculation of s_c: $s_m = 0.265 f_p / n_B = 14.05 \Rightarrow s_c = \sqrt{s_w^2 + s_m^2} = 14.7$

$$I_{total} = 2\left(\frac{f_p}{2p s_c}\right)^4 = 2168 = 33.4 \text{dB}$$ or, alternately: $$\frac{1}{I_{total}} = \frac{1}{I_{c2}} + \frac{1}{I_{s2}}$$

Coherent and Noncoherent Pulse Trains

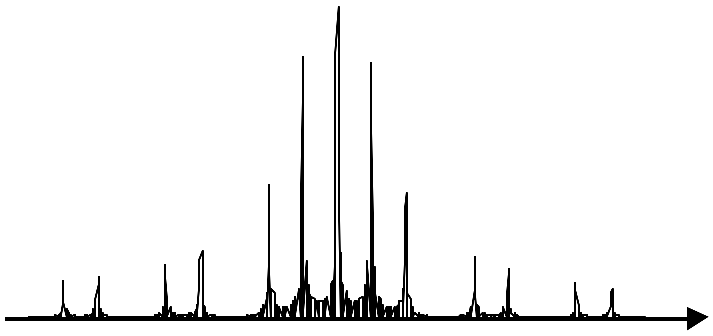
A coherent pulse train is one that is "cut" from a parent sinusoid. The waveforms A and B overlap exactly. For a noncoherent pulse train, the initial value of the carrier sinusoid for each pulse is essentially random.



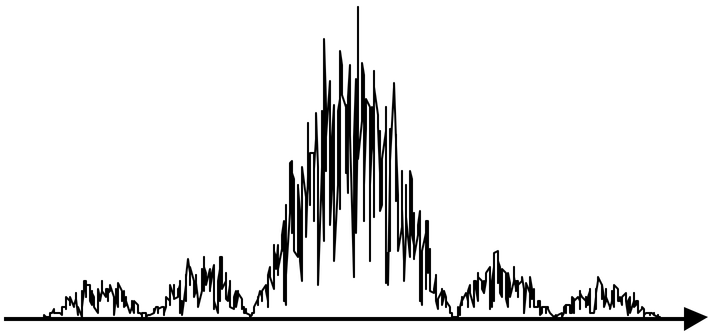
Noncoherent Pulse Train Spectrum (1)

Random phases between pulses cause the spectrum to smear. Sharp doppler lines do not exist. However, if the target velocity is high enough and its modulation of the returned pulse is strong, the target can be detected.

SPECTRUM OF A COHERENT PULSE TRAIN



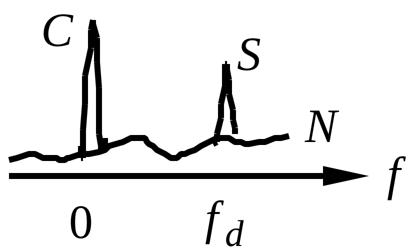
SPECTRUM OF A NON-COHERENT PULSE TRAIN



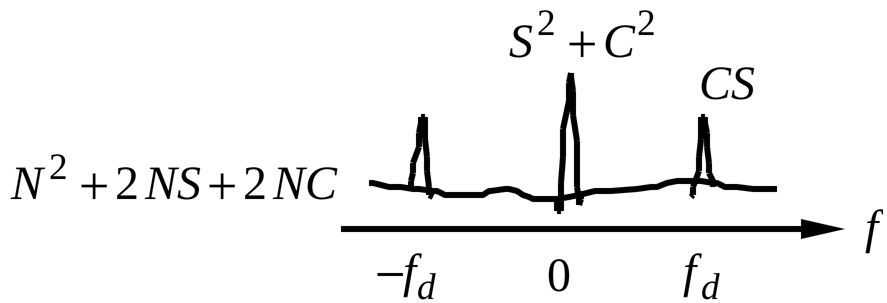
Noncoherent Pulse Train Spectrum (2)

Video spectra:

Coherent



Noncoherent



The convolution of signal and clutter with noise gives the power

$$N^2 + 2NS + 2NC$$

The signal-to-noise ratio is

$$\frac{S}{N} = \frac{2SC}{N^2 + 2NS + 2NC}$$

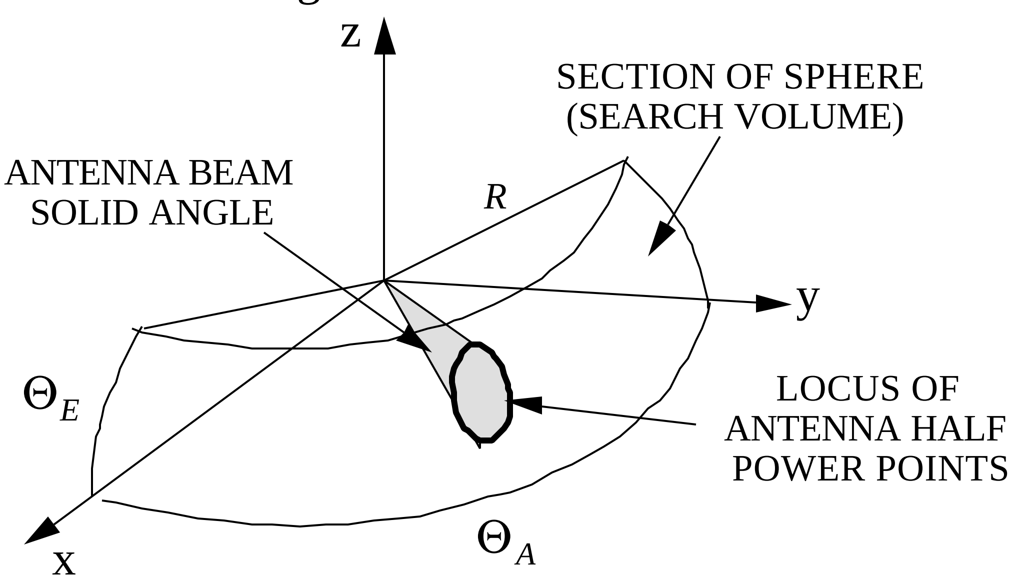
Note that clutter must be present or the SNR is zero.

Search Radar Equation (1)

Search radars are used to acquire targets and then hand them off to a tracking radar. (A multifunction radar can perform both tasks.) Search requires that the antenna cover large volumes of space (solid angles) in a short period of time. This implies:

1. fast antenna scan rate if the beam is narrow
2. large antenna beamwidth if a slow scan is used

Consider a volume search at range R:



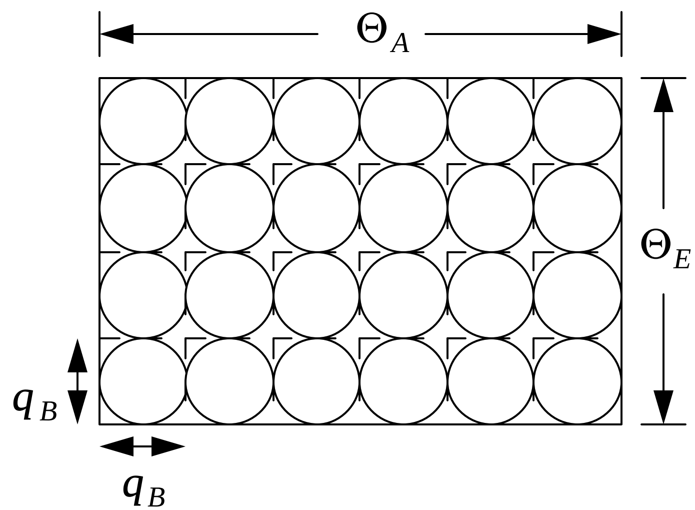
Search Radar Equation (2)

Search volume:
$$\Omega_s = \frac{A_s}{R^2} = \frac{1}{R^2} \int_{\Theta_{E1}}^{\Theta_{E2}} \int_{\Theta_{A1}}^{\Theta_{A2}} R^2 \sin q \, dq \, df$$

where $(\Theta_{E2}, \Theta_{E1})$ are the elevation scan limits and $(\Theta_{A2}, \Theta_{A1})$ are the azimuth scan limits. If the volume searched is near the horizon then $q \approx 90^\circ$ and

$$\Omega_s \approx (\Theta_{E2} - \Theta_{E1})(\Theta_{A2} - \Theta_{A1}) \equiv \Theta_E \Theta_A$$

Assume antenna beams with circular cross section. "Plan view" of the scan region:



Search Radar Equation (3)

Number of beam positions required

$$N_B \approx \frac{\text{frame area}}{\text{beam area}} = \frac{\Theta_E \Theta_A}{q_B^2}$$

The target can only lie in one beam. The <u>time on target</u> t_{ot}, or <u>dwell time</u>, is the time spent at each beam position

$$t_{\text{ot}} = \frac{t_f}{N_B} = \frac{t_f q_B^2}{\Theta_E \Theta_A} \equiv \frac{n_B}{f_p}$$

where n_B is the number of pulses transmitted per beam position. Assuming that all pulses are integrated, the search radar equation becomes

$$\text{SNR} = \frac{P_r}{N_o} = \frac{P_t G_t A_{er} s\, n_B}{(4p)^2 k \mathrm{T}_s B_n R^4 L}$$

Now use

$$t_{\text{ot}} = t_f / N_B,\ B_n t \approx 1,\ G_t = \frac{4p(p D^2/4)}{l^2},\ q_B \approx \frac{l}{D},\ f_p = \frac{n_B}{t_{ot}},\ \text{and } P_t = \frac{P_{\text{av}}}{t\ f_p}$$

Search Radar Equation (4)

Search radar equation becomes

$$\mathrm{SNR} = \frac{P_r}{N_o} = \frac{P_{\mathrm{av}} A_{er} s \ t_f}{16 k \mathrm{T}_s R^4 L N_B q_B^2}$$

(Note: Skolnik has $4p$ rather than 16 due to $q_B \approx 0.88 l / D$ vs l / D). Using $N_B q_B^2 = \Theta_E \Theta_A$ and rearranging gives the search detection range

$$R_{\max} = \sqrt[4]{\frac{P_{\mathrm{av}} A_{er} s \ t_f}{16 k \mathrm{T}_s L \Theta_E \Theta_A \ \mathrm{SNR}}}$$

Points to note:

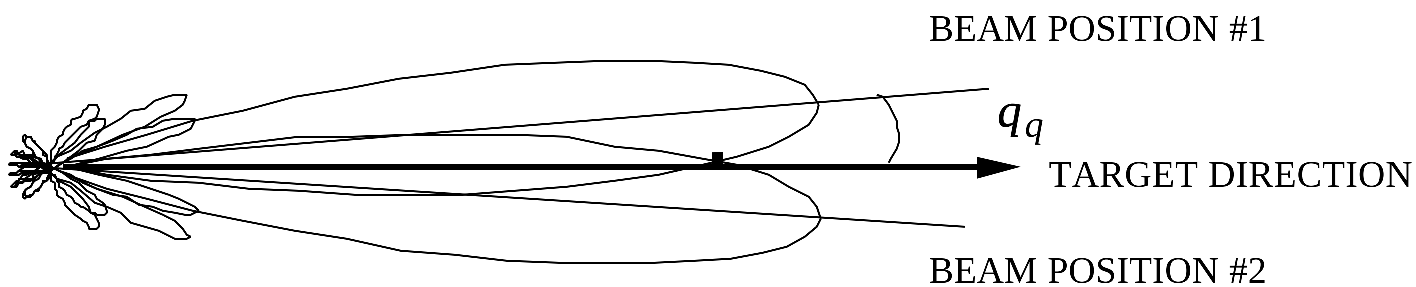
1. independent of frequency (wavelength)
2. for given values of $t_f / \Theta_E \Theta_A$ and s, the range primarily depends on the product $P_{\mathrm{av}} A_{er}$
3. $t_f / \Theta_E \Theta_A$ must be increased to increase R
4. note that coherent integration of n_B pulses has been assumed

Common search patterns: raster, spiral, helical, and nodding (sinusoidal)

Radar Tracking (1)

Target tracking is achieved by positioning the radar antenna boresight (direction reference with respect to the gain pattern). Several techniques can be used:

1. Sequential lobing: The beam is switched between positions #1 and #2. q_q is the squint angle. If the target return is constant and located at the bisecting angle of the two beam maxima, then the received power will be the same in both positions.

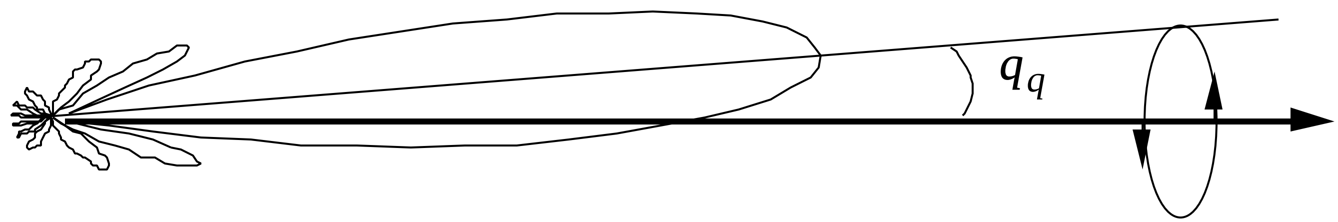


   A error in the direction estimate occurs if the target RCS is not constant (which is always the case).

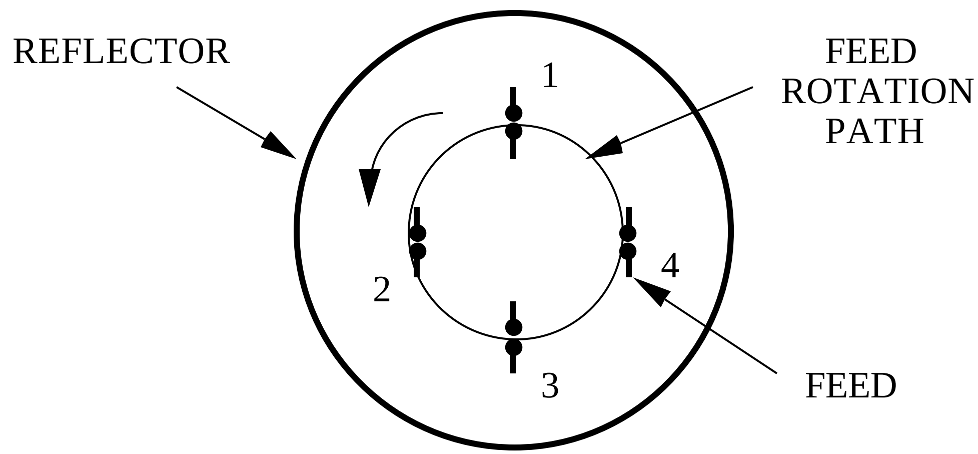
2. Conical scan: The antenna beam is squinted a small amount and then rotated around the reference. If the target is in the reference direction the received power is constant. If not the received power is modulated. The modulation can be used to generate an error signal to correct the antenna pointing direction.

Radar Tracking (2)

Conical scan (continued)



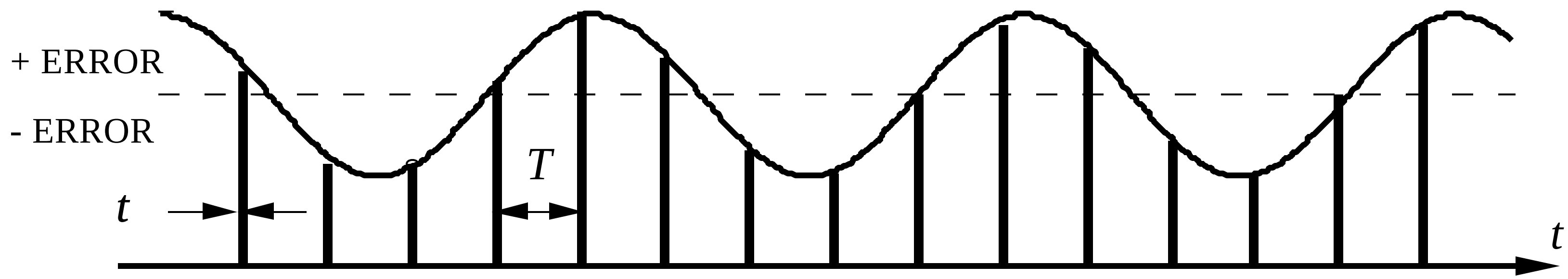
Note that if the antenna is physically rotated the polarization also rotates. This is undesirable. Fixed polarization is provided by a <u>nutating feed</u>.



Radar Tracking (3)

Conical scan (continued)

If the target is not centered on the axis of rotation then there is a modulation of the received power



Conical scan problems:

1. Jet turbines rotate at about the same frequency as the upper limit of antenna rotation (2400 rpm). Propellers are at the lower end of the antenna rotation limit (100 rpm).
2. Long ranges are a problem. The round trip time of transit is comparable to the antenna rotation.
3. Pulse-to-pulse RCS variations are a problem.

Gain Control

The dynamic range of received signals can exceed the dynamic range of the receiver

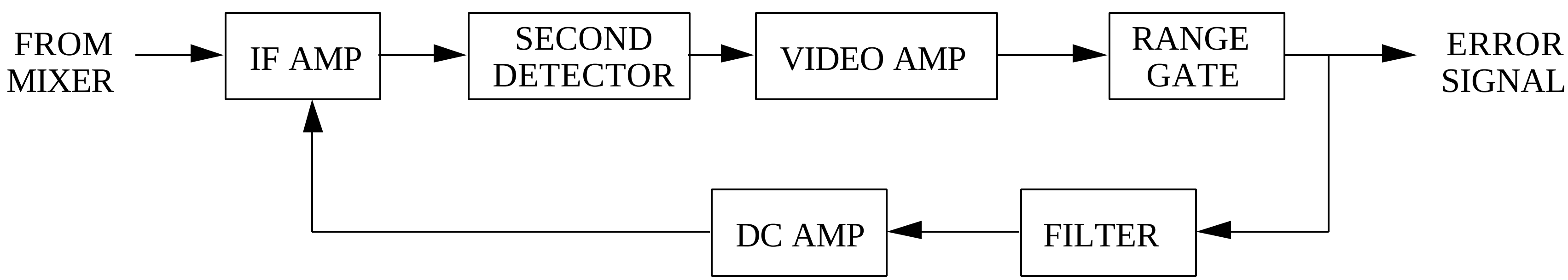
close in targets ⇒ large return above receiver saturation level (any scanning modulation is lost)
far out targets ⇒ small return below noise level

The dynamic range can be extended using gain controls:

Manual gain control (MGC): The operator adjusts the receiver to match the dynamic range of the display.

Automatic gain control (AGC): The signal from the target in a range gate is kept at a constant level

AGC circuit:

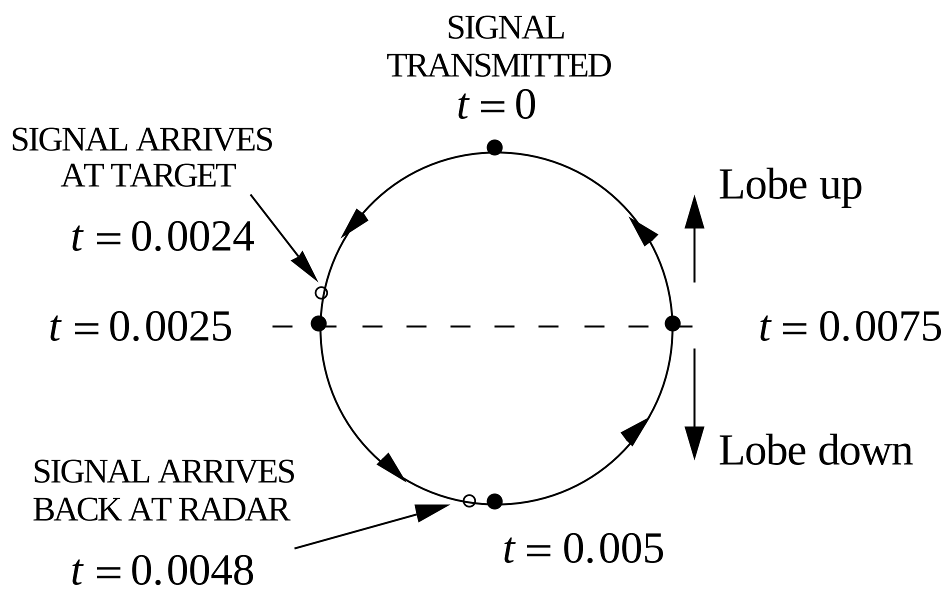


Example

Conical scan antenna with a rotation rate of 100 Hz (0.01 sec/revolution) operates at a range of 460 miles. The one-way transit time of a pulse is

$$t = R/c = (460)(1000)/\left[(0.62)\left(3\times10^{8}\right)\right] = 0.0024 \text{ sec}$$

The antenna rotates 90° in 0.01/4 = 0.0025 sec. The arrival time of a pulse is illustrated below:



Four pulses are generally required (top, bottom, left, and right). For the case of the target left or right, the transit time is approximately equal to half of the rotation rate, and therefore the effect of the scan is cancelled. There is no modulation of the return.

Example

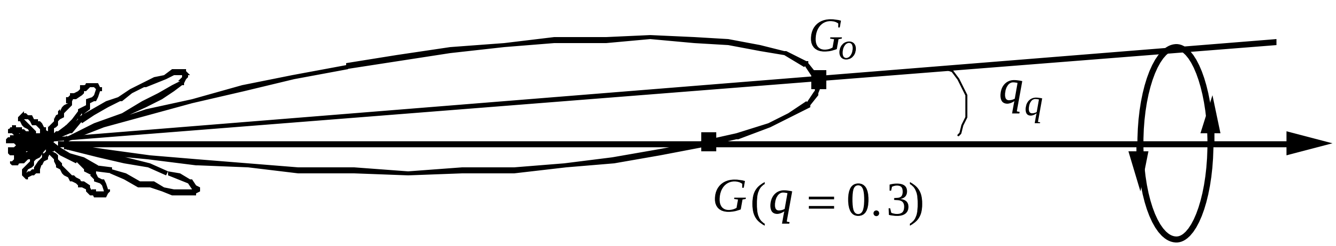
A conical scan radar has a gaussain shaped main beam gain approximated by

$$G(q) = G_o e^{-Kq^2/q_B^2}$$

where $q_B = 1^\circ$ is the HPBW and $K = 4\ln(2) = 2.773$. The <u>crossover loss</u> is the reduction in gain due to the fact that the target is tracked off of the beam peak. Assume that the beam is squinted at an angle $q_q = 0.3^\circ$. The pattern level at the squint angle is

$$G(q_q) = G_o \exp\left\{\left(-2.773(0.3^\circ)^2/(1^2)\right)\right\} = 0.779 G_o$$

The crossover loss is $10\log\left(\dfrac{0.779 G_o}{G_o}\right) = -1.084$ dB

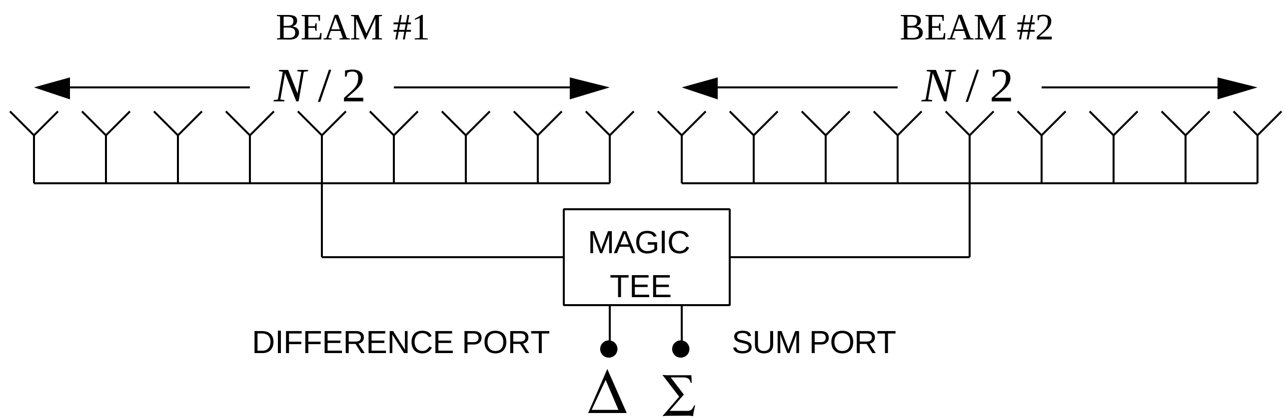


Monopulse Tracking (1)

Pulse to pulse variations in the target RCS leads to tracking inaccuracies. We want range and angle information with a single pulse, i.e., monopulse (also called simultaneous lobing). There are two types:

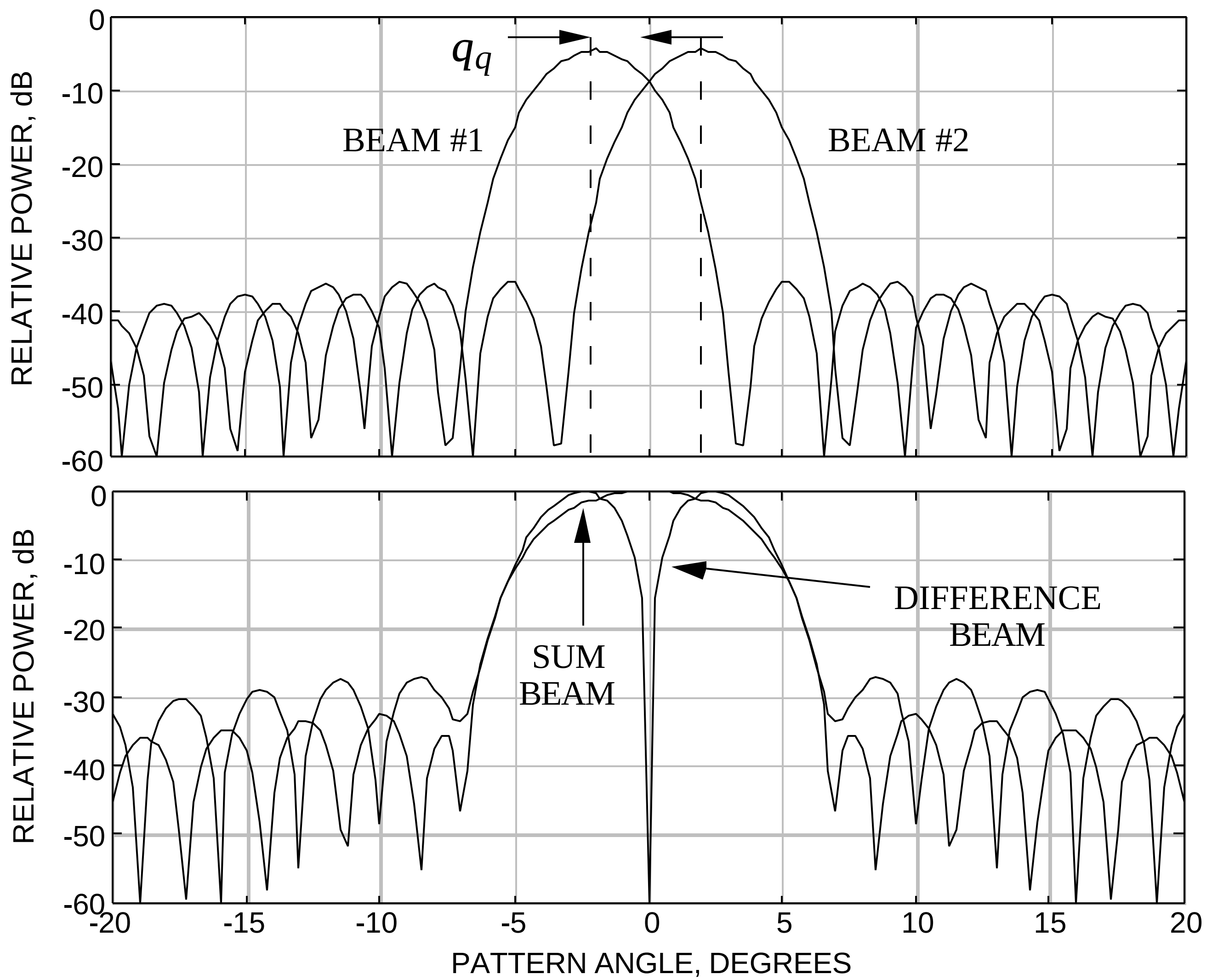
1. Amplitude comparison monopulse:

Two antenna beams are generated at small positive and negative squint angles. The outputs from the two beam ports are added to form a sum beam and subtracted to form a difference beam. The sum beam is used on transmit and receive; the difference beam is only used on receive.



Monopulse beamforming is implemented using a magic tee.

Monopulse Tracking (2)



Monopulse Tracking (3)

Tracking is done by processing the difference to sum voltage ratio:

$$\frac{\Delta}{\Sigma} = \frac{\text{difference voltage}}{\text{sum voltage}} = \frac{\Delta_I + j\Delta_Q}{\Sigma_I + j\Sigma_Q}$$

Now,

$$\mathrm{Re}\{\Delta/\Sigma\} = \frac{\Delta_I \Sigma_I + \Delta_Q \Sigma_Q}{\Sigma_I^2 + \Sigma_Q^2} = \frac{|\Delta|}{|\Sigma|}\cos \boldsymbol{d}$$

$$\mathrm{Im}\{\Delta/\Sigma\} = \frac{\Delta_Q \Sigma_I - \Delta_I \Sigma_Q}{\Sigma_I^2 + \Sigma_Q^2} = \frac{|\Delta|}{|\Sigma|}\sin \boldsymbol{d}$$

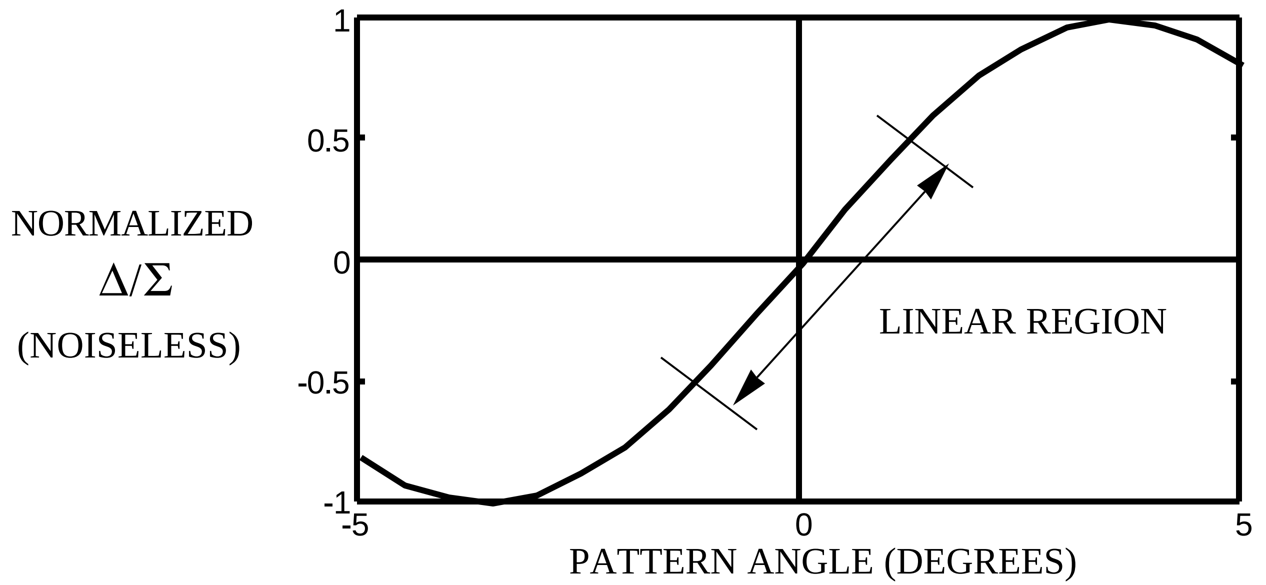
where $\boldsymbol{d}$ is the relative phase between the sum and difference channels. Usually only $\mathrm{Re}\{\Delta/\Sigma\}$ is processed because:

1. It has the required sign information: + ratio on positive side of null; - on negative side.
2. The target only contributes to $\mathrm{Re}\{\Delta/\Sigma\}$; noise, interference, etc. contribute to both terms equally.

See Fig. 5.9 in Skolnik for implementation. I and Q processing can be done after the amplifier.

Monopulse Tracking (4)

Plot of typical Δ / Σ in the vicinity of the null. (The slope depends on the antenna beamwidths and squint.)



In the linear region:

$$\Delta / \Sigma \approx Kq$$

K is the monopulse slope constant. The function Δ / Σ can be used to generate an error signal to place the difference beam null on the target.

Monopulse Tracking (5)

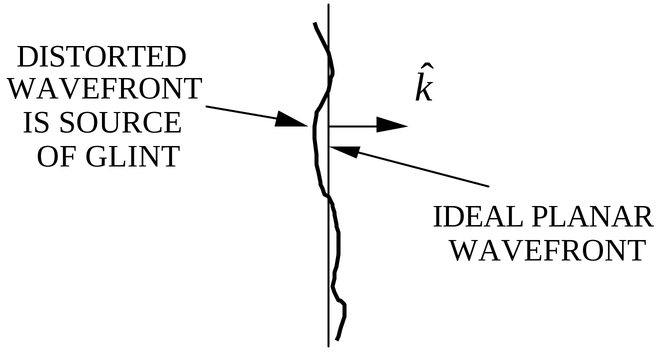
Sources of monopulse tracking error:

1. Antenna errors: null shift and null filling due to antenna illumination errors.
2. Thermal noise: RMS angle error for a single target, high SNR, with the target on boresight (i.e., the target would be in the null of the ideal antenna)

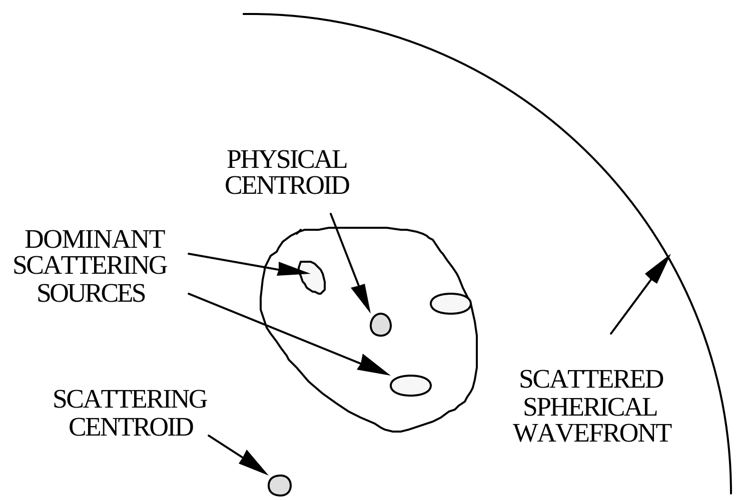
$$s_{q_t} = \frac{1}{K\sqrt{\mathrm{SNR}}}$$

3. Target glint: glint refers to the distortion of the wavefront scattered from the target due to environmental and interference effects. When the wavefront is distorted, the apparent target direction can differ from the actual target direction.

NONPLANAR WAVEFRONT



SPHERICAL SOURCE DISPLACEMENT

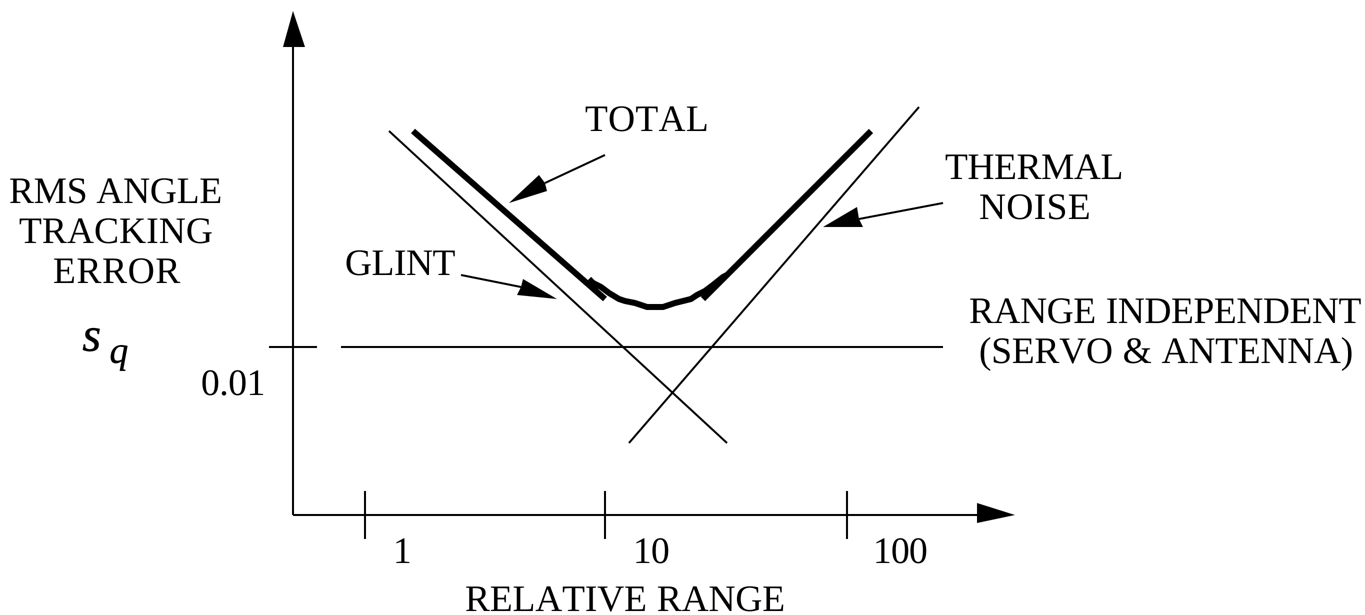


Monopulse Tracking (6)

RMS angle error due to target glint is approximately given by the empirical formula

$$\sigma_{\theta_g} \approx 0.7 \tan\left(\frac{L/2}{R}\right) \approx 0.35 \frac{L}{R}$$

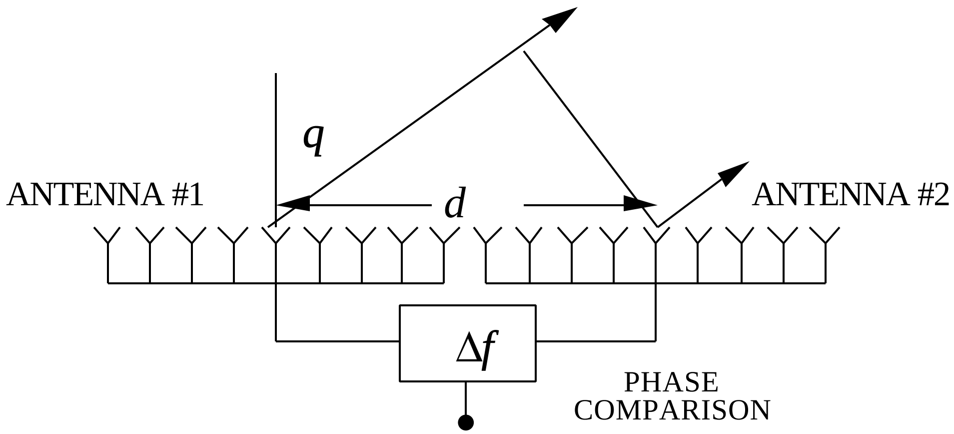
where L is the target extent (for example, the length or wingspan of an aircraft), and R is the range to the target.



Monopulse Tracking (7)

1. Phase comparison monopulse (interferometer radar):

The phase difference between two widely spaced antennas is used to determine the angle of arrival of the wavefront.



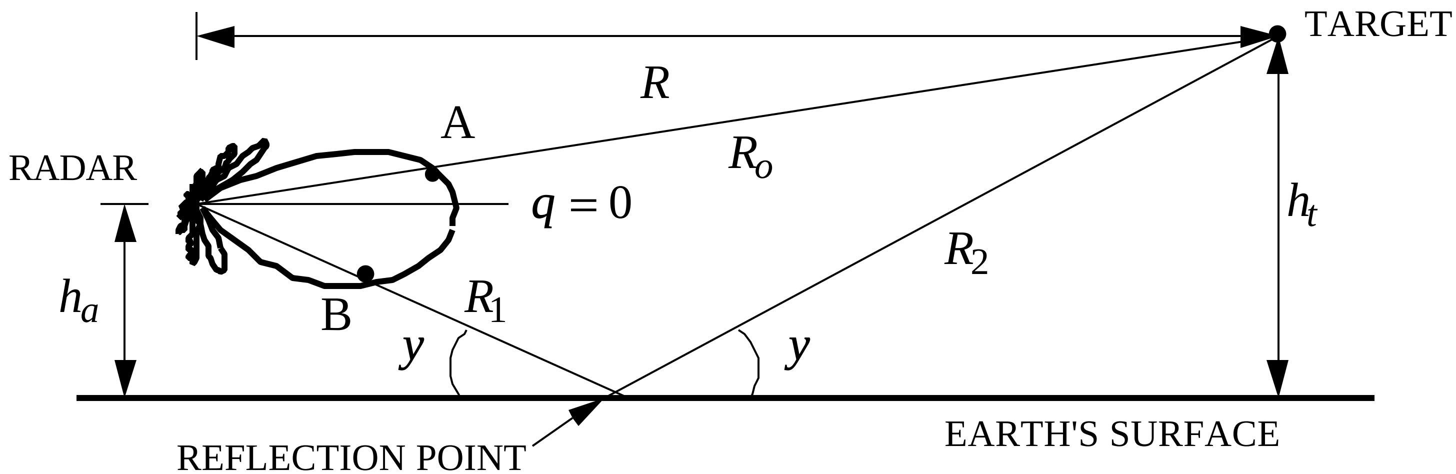
For a plane wave arriving from a direction q, the phase difference between antennas #1 and #2 can be used to determine q:

$$\Delta f = kd \sin q \approx kdq \quad \Rightarrow \quad q \approx \Delta f \,/(kd)$$

Problems: 1. ambiguities due to grating lobes because the antennas are widely spaced
2. tight phase tolerances must be maintained on the antenna
3. thermal and servo noise are sources of error

Low Angle Tracking (1)

When a radar and target are both operating near the surface of the earth, multipath (multiple reflections) can cause extremely large angle errors. Assume a flat earth:



At low altitudes the reflection coefficient is approximately constant ($\Gamma \approx -1$) and $G_D(q_A) \approx G_D(q_B)$. The difference between the direct and reflected paths is:

$$\Delta R = \underbrace{(R_1 + R_2)}_{\text{REFLECTED}} - \underbrace{R_o}_{\text{DIRECT}}$$

Low Angle Tracking (2)

The total signal at the target is:

$$E_{\mathrm{tot}} = \underbrace{E_{\mathrm{ref}}}_{\text{REFLECTED}} + \underbrace{E_{\mathrm{dir}}}_{\text{DIRECT}} = E(q_A) + \Gamma\, E(q_B) e^{-jk\Delta R}$$

From the low altitude approximation, $E_{\mathrm{dir}} = E(q_A) \approx E(q_B)$ so that

$$E_{\mathrm{tot}} \approx E_{\mathrm{dir}} + \Gamma\, E_{\mathrm{dir}} e^{-jk\Delta R} = E_{\mathrm{dir}} \left| \underbrace{\left[1 + \Gamma\, e^{-jk\Delta R}\right]}_{\equiv F,\ \text{PATH GAIN FACTOR}} \right|$$

The path gain factor takes on the values $0 \le F \le 2$. If $F = 0$ the direct and reflected rays cancel (destructive interference); if $F = 2$ the two waves add (constructive interference).

An approximate expression for the path difference:

$$R_o = \sqrt{R^2 + (h_t - h_a)^2} \approx R + \frac{1}{2}\frac{(h_t - h_a)^2}{R}$$

$$R_1 + R_2 = \sqrt{R^2 + (h_t + h_a)^2} \approx R + \frac{1}{2}\frac{(h_t + h_a)^2}{R}$$

Low Angle Tracking (3)

Therefore,

$$\Delta R \approx \frac{2h_a h_t}{R}$$

and

$$|F| = \left|1 - e^{-jk2h_a h_t / R}\right| = \left|e^{jkh_a h_t / R}\left(e^{-jkh_a h_t / R} - e^{jkh_a h_t / R}\right)\right| = 2\left|\sin\left(kh_a h_t / R\right)\right|$$

Incorporate the path gain factor into the RRE:

$$P_r \propto |F|^4 = 16 \sin^4\left(\frac{kh_t h_a}{R}\right) \approx 16\left(\frac{kh_t h_a}{R}\right)^4$$

The last approximation is based on

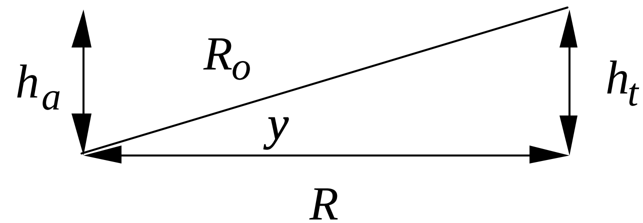
$$h_a << R \text{ and } h_t << R.$$

Finally, the RRE can be written as

$$P_r = \frac{P_t G_t G_r \, l^2 s}{(4p)^3 R^4} \, |F|^4 \approx \frac{4p \, P_t G_t G_r s \, (h_t h_a)^4}{l^2 R^8}$$

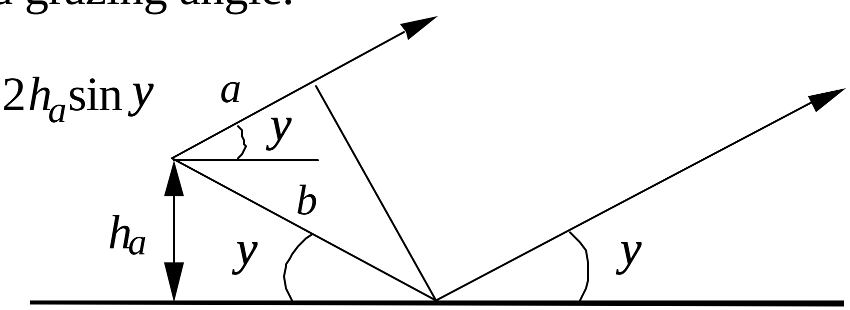
Low Angle Tracking (4)

Two other forms of F are often used. Define y as the <u>elevation angle</u> from the ground, $\tan y = h_t / R$. Therefore $|F|^4 = 16\sin^4\left(kh_a \tan y\right)$



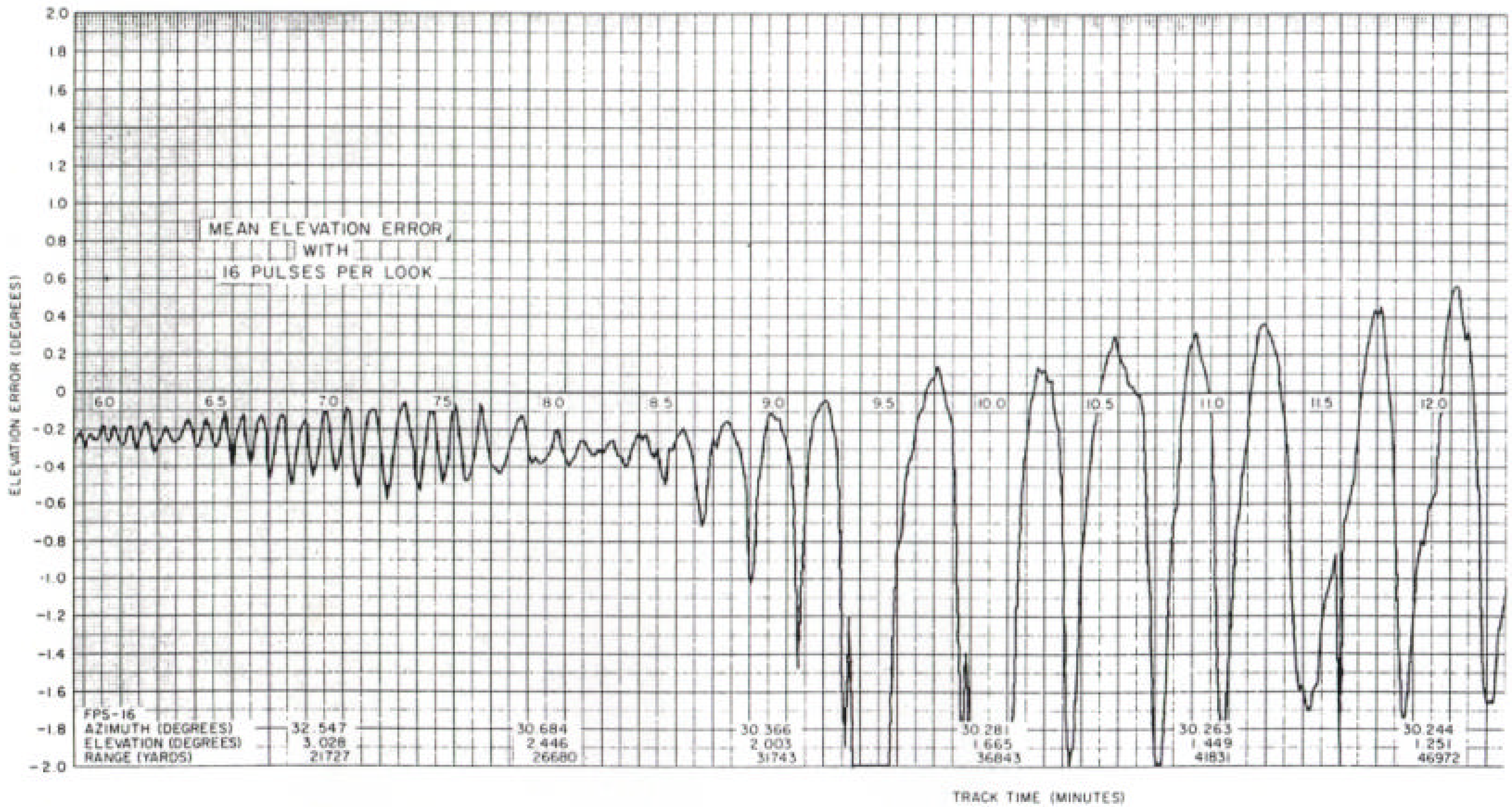
For the second form, assume the far-field parallel ray approximation is valid. Then y is both the elevation angle and grazing angle.

$$\Delta R = b - a = 2h_a \sin y$$



Therefore, $|F|^4 = 16\sin^4\left(\dfrac{2p\, h_a \sin y}{l}\right)$ with nulls at $\sin y_n = \dfrac{n\,l}{2h_a}, n = 0,1,2,\ldots$

Tracking Error Due to Multipath



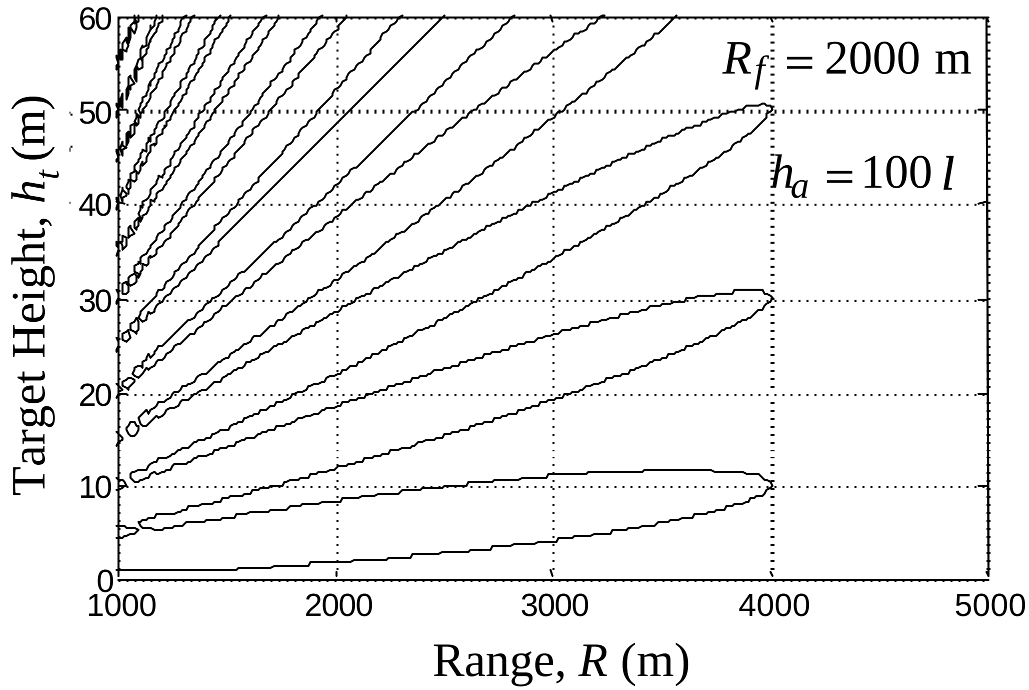
(ZENITH) (Fig. 4.18 in Skolnik) (HORIZON)

Low Angle Tracking (5)

Methods of displaying the received power variation:

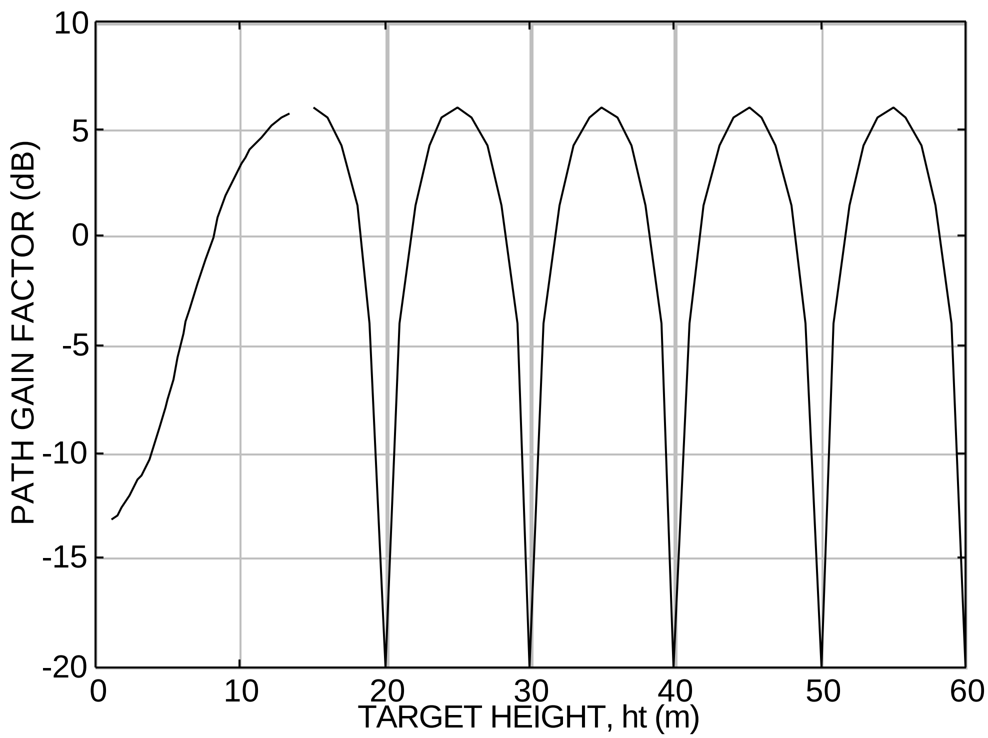
1. Coverage diagram: Contour plots of $|F|$ in dB vs h_t, R normalized to a reference range R_f. Contours of power equal to that of the free space reference range are plotted.

$$|F| = \left| 2\left(\frac{R_f}{R}\right)\sin\left(kh_a \tan\psi\right) \right|$$



Low Angle Tracking (6)

2. Height-gain curves: Plots of $|F|$ in dB vs h_t at a fixed range. The constructive and destructive interference as a function of height can be identified. At low frequencies the periodicity of the curve at low heights can be destroyed by the ground wave



Atmospheric Refraction (1)

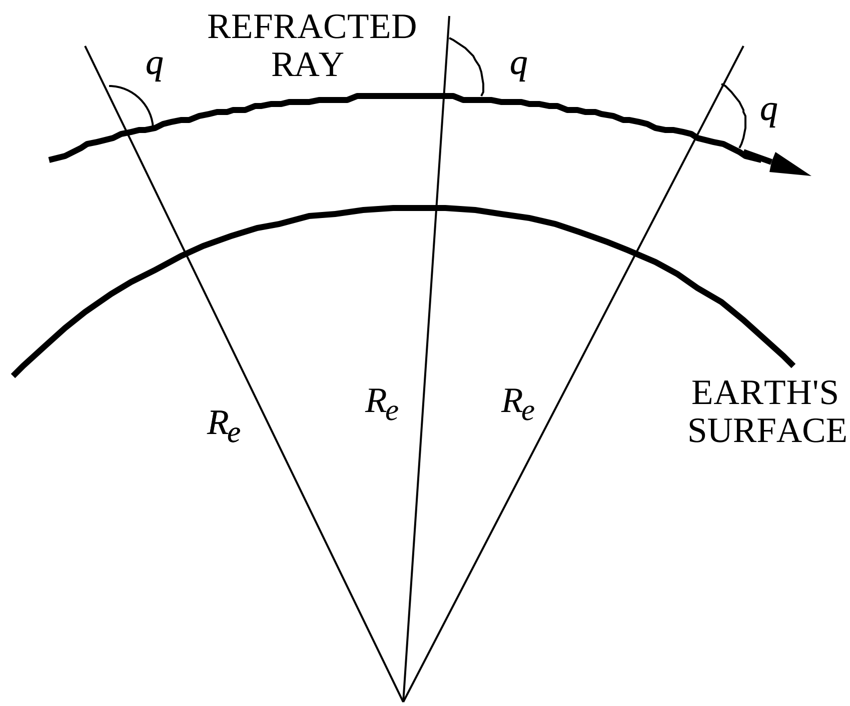
Refraction by the atmosphere causes waves to be bent back towards the earth's surface.
The ray trajectory is described by the equation: $n\,R_e \sin q = \text{CONSTANT}$
Two ways of expressing the index of refraction in the troposphere:

1. $n = 1 + c r / r_{\text{SL}}$
 + HUMIDITY TERM

R_e = 6378 km = earth radius
$c \approx 0.00029 =$ Gladstone-Dale constant
$r,\ r_{SL}$ = mass densities at altitude and sea level

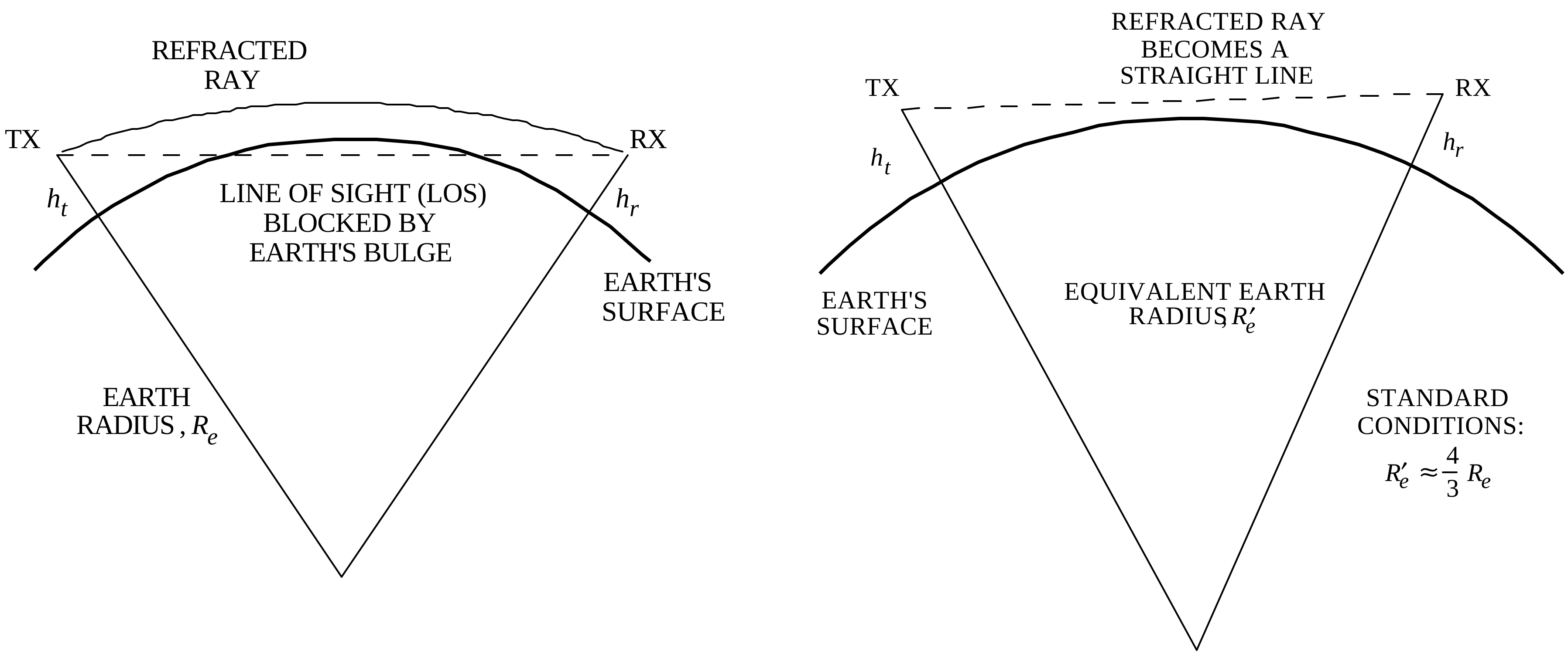
2. $n = 77.6\,p/\mathrm{T} + 7.73\times10^{5}\,e/\mathrm{T}^{2}$

$p =$ air pressure (millibars)
T = temperature (K)
$e =$ partial pressure of water vapor (millibars)



Atmospheric Refraction (2)

Refraction of a wave can provide a significant level of transmission over the horizon. A refracted ray can be represented by a straight ray if an equivalent earth radius is used.



Atmospheric Refraction (3)

Distance from the transmit antenna to the horizon: $R_t = \sqrt{(R_e' + h_t)^2 - (R_e')^2}$

but $R_e' >> h_t$ so that

$$R_t \approx \sqrt{2 R_e' h_t}$$

similarly,

$$R_r \approx \sqrt{2 R_e' h_r}$$

The radar horizon is the sum

$$R_{RH} \approx \sqrt{2 R_e' h_t} + \sqrt{2 R_e' h_r}$$